

# $t \rightarrow c\gamma$ and $t \rightarrow cg$ in Warped Extra Dimensions

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## Abstract

In this work, we calculate the top quark rare decays  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  in the framework where the standard model is embedded in a warped extra dimension with the custodial symmetry  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times P_{LR}$ . Adopting reasonable assumptions on the parameter space, we numerically find the branching ratios of  $t \rightarrow c\gamma$  exceeding  $10^{-6}$  and that of  $t \rightarrow cg$  exceeding  $10^{-5}$  respectively, which can be detected in near future.

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## I. INTRODUCTION

Top quark plays a special role in the Standard Model(SM) and holds great promise in revealing the secret of new physics beyond the SM. And the study of the rare top quark decays have long been a subject of intense theoretical and experimental study. Among those rare processes, the flavor-changing neutral current (FCNC) decays  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  deserve special attention, since the branching ratios(BRs) of those rare processes are strongly suppressed in the SM.

On the theoretical aspect, the authors of Ref.[1] gives a general expression for the one-loop fermion-neutral boson coupling keeping all masses and momenta. In the framework of the SM, the BRs of top quark FCNC  $t \rightarrow c\gamma$  is of the order  $10^{-13}$  and that of  $t \rightarrow cg$  is of the order  $10^{-11}$  [2, 3]. In extensions of the SM, the BRs for FCNC top decays can be orders of magnitude larger. For example in two Higgs doublet models  $\text{Br}(t \rightarrow c\gamma) \sim 10^{-7}$ ,  $\text{Br}(t \rightarrow cg) \sim 10^{-5}$  can be achieved [4], and in supersymmetric models with R parity conservation these branching ratios can reach  $\text{Br}(t \rightarrow c\gamma) \sim 10^{-6}$ ,  $\text{Br}(t \rightarrow cg) \sim 10^{-5}$  [5, 6]. The authors of Ref.[7] discuss the process  $t \rightarrow c\gamma$  in a model with a single universal extra dimension which is compactified gauge, and get the branching fraction  $\text{Br}(t \rightarrow cg) \sim 10^{-10}$ . In Ref.[8], the author computed the BRs for  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  in minimal extensions of the SM, where the additional vector-like up and down quarks singlets break the unitarity of the  $3 \times 3$  Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The running LHC is a top-quark factory, and provides a great opportunity to seek out top-quark rare decays. Given the annual yield of 80 million  $t\bar{t}$  events plus 34 million single-top events, one may hope to search for rare decays with a branching fraction as small as  $10^{-6}$ [9]. Then the experimental observation on LHC will bring a tremendous improvement in our knowledge of top quark properties.

Models with a warped extra dimension, also called Randall-Sundrum (RS) models [10, 11], where all SM fields are allowed to propagate in the bulk, offer natural solutions to many outstanding puzzles of contemporary particle physics. In addition to providing a geometrical solution to the hierarchy problem related to the vast difference between the Planck scale

and the electroweak (EW) scale, they also allow to naturally generate hierarchies in fermion masses and weak mixing angles [12, 13], suppress FCNC interactions [14–16], construct realistic models of EW symmetry breaking [17–21] and achieve gauge coupling unification [22, 23].

A necessary condition for direct signals of RS models at the LHC is the existence of Kaluza-Klein (KK) modes with  $\mathcal{O}(1\text{TeV})$  masses. But early studies of EW precision observables [16, 24] have shown that with the SM gauge group  $SU(2)_L \times U(1)_Y$  in the bulk, the EW precision observables, for example the experimental data on  $S$ ,  $T$  parameters and the well-measured  $Z\bar{b}_L b_L$  coupling [25–28], generally require that the exciting KK modes are heavier than 10 TeV and exceed the reach of colliders running now. To solve this problem, literature [29, 30] enlarges the gauge group in the bulk to  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times P_{LR}$ . The presence of new light KK modes necessary to solve the  $Z\bar{b}_L b_L$  problem, implies significant contributions to the  $T$  parameter at the one loop level [31]. With an appropriate choice of quark bulk mass parameters, an agreement with the EW precision data in the presence of light KK modes can be obtained [32, 33]. Actually, the EW precision observables are consistent with the light fermion KK modes with masses even below 1TeV while the masses of KK gauge bosons are forced to be at least 2 – 3TeV to be consistent with experimental data on the parameter  $S$ .

It is well known that all virtual KK excitations contribute their corrections to theoretical predictions on the physical quantities at EW scale, and those theoretical corrections should be summed over infinite KK modes in principle [34–36]. In this paper, we sum over the infinite series of KK modes using the method in Ref. [37], and analyze the corrections from exciting KK modes to the top-quark decay  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  in the scenario with a warped extra dimension and the custodial symmetry.

This paper is composed of the sections as follows. In section II, we present the main ingredients of the SM extension with a warped extra dimension and the custodial symmetry. And we list some useful formulas for summing over infinite series of KK modes. In section III, we present the theoretical calculation on the  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  processes. Section IV is devoted to the numerical analysis and discussion. In section V, we deliver our conclusions. The relevant nontrivial couplings approached to the order  $\mathcal{O}(\mu_{EW}^2/\Lambda_{KK}^2)$  are in appendix,

where  $\Lambda_{KK}$  denotes the energy scale of low-lying KK excitations and  $\mu_{EW}$  denotes the EW energy scale.

## II. A WARPED EXTRA DIMENSION WITH CUSTODIAL PROTECTION AND SUMMING OVER INFINITE SERIES OF KK MODES

Models with extra dimensions have attracted great attention in recent years as they offer new perspectives on challenging problems in modern physics. It was demonstrated by Randall and Sundrum that a small but warped extra dimension provides an elegant solution to the gauge hierarchy problem [10, 38]. In the RS scenario, four dimensional (4D) Minkowskian space-time is embedded into a slice of five dimensional (5D) anti de-Sitter ( $ADS_5$ ) space with curvature  $k$ . The fifth dimension is a  $S^1/Z_2 \times Z'_2$  orbifold of size  $r$  labeled by a coordinate  $\phi \in [-\pi, \pi]$ , such that the points  $(x^\mu, \phi)$ ,  $(x^\mu, \pi - \phi)$ ,  $(x^\mu, \pi + \phi)$  and  $(x^\mu, -\phi)$  are identified all. The corresponding metric of the non-factorizable RS geometry is given by

$$ds^2 = e^{-2\sigma(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu - r^2 d\phi^2, \quad \sigma(\phi) = kr|\phi|, \quad (1)$$

where  $x^\mu$  ( $\mu = 0, 1, 2, 3$ ) denote the coordinates on the 4D hyper-surfaces of constant  $\phi$  with metric  $\eta_{\mu\nu} = (1, -1, -1, -1)$ , and  $e^\sigma$  is called the warp factor. Two branes are located on the orbifold fixed points  $\phi = 0$  and  $\phi = \pi/2$ , respectively. The brane on  $\phi = 0$  is called Planck or ultra-violet (UV) brane, and the brane on  $\phi = \pi/2$  is called TeV or infra-red (IR) brane. The parameters  $k$  and  $1/r$  are assumed to be of order the fundamental Planck scale  $M_{Pl}$  and choosing the product  $kr \simeq 24$ , one gets the inverse warp factor

$$\epsilon = \frac{\Lambda_{IR}}{\Lambda_{UV}} \equiv e^{-kr\pi/2} \simeq 10^{-16}, \quad (2)$$

which explains the hierarchy between the EW and Planck scale naturally. Meanwhile, the warp factor also sets the mass scale for the low-lying KK excitations

$$\Lambda_{KK} \equiv k\epsilon = ke^{-kr\pi/2} = \mathcal{O}(1\text{TeV}). \quad (3)$$

In the EW sector, we consider an  $SU(2)_L \times SU(2)_R \times U(1)_X \times P_{LR}$  gauge symmetry on a slice of  $\text{AdS}_5$ , where  $P_{LR}$  is the discrete symmetry interchanging the two  $SU(2)$  groups. This means for instance that  $g_{5L} = g_{5R} \equiv g_5$ . The gauge group is broken by boundary conditions (BCs) on the UV brane to the SM gauge group, i. e.

$$SU(2)_L \times SU(2)_R \times U(1)_X \times P_{LR} \xrightarrow{UV \text{ brane}} SU(2)_L \times U(1)_Y. \quad (4)$$

This breakdown is achieved by the following assignment of BCs

$$\begin{aligned} &W_{L,\mu}^{1,2,3}(++), B_\mu(++), W_{R,\mu}^{1,2}(-+), Z_{X,\mu}(-+), \quad (\mu = 0, 1, 2, 3), \\ &W_{L,5}^{1,2,3}(--), B_5(--), W_{R,5}^{1,2}(+-), Z_{X,5}(+-). \end{aligned} \quad (5)$$

where the first (second) sign is the BC on the UV (IR) brane:  $+$  stands for a Neumann BC and  $-$  stands for a Dirichlet BC. The third component of  $SU(2)_R$  gauge fields  $W_{R,M}^3$  and the  $U(1)_X$  gauge field  $\tilde{B}_M$  are expressed in terms of the neutral gauge fields  $Z_{X,M}$  and  $B_M$  as

$$W_{R,M}^3 = \frac{g_5 Z_{X,M} + g_{5X} B_M}{\sqrt{g_5^2 + g_{5X}^2}}, \quad \tilde{B}_M = -\frac{g_{5X} Z_{X,M} - g_5 B_M}{\sqrt{g_5^2 + g_{5X}^2}}, \quad (M = 0, 1, 2, 3, 5), \quad (6)$$

where  $g_{5X}$  is the 5D gauge coupling of  $U(1)_X$ .

To further proceed it will be useful to follow [39] and define the fields

$$\begin{aligned} A_A &= \frac{\sqrt{g_5^2 + g_{5X}^2} B_A + g_{5X} W_{L,A}^3}{\sqrt{g_5^2 + 2g_{5X}^2}}, \\ Z_A &= \frac{-g_{5X} B_A + \sqrt{g_5^2 + g_{5X}^2} W_{L,A}^3}{\sqrt{g_5^2 + 2g_{5X}^2}}, \\ W_{L,A}^\pm &= \frac{1}{\sqrt{2}} \left( W_{L,A}^1 \mp i W_{L,A}^2 \right), \\ W_{R,A}^\pm &= \frac{1}{\sqrt{2}} \left( W_{R,A}^1 \mp i W_{R,A}^2 \right). \end{aligned} \quad (7)$$

In order to break down the electroweak symmetry, a Higgs boson is introduced that is localised either on or near the IR brane, transforming as a self-dual bidoublet of  $SU(2)_L \times SU(2)_R$ , and transforms as a singlet with charge  $Y_H = 0$  under the gauge group  $U(1)_X$ :

$$H = \begin{pmatrix} -i\pi^+/\sqrt{2} & -(h^0 - i\pi^0)/2 \\ (h^0 + i\pi^0)/2 & i\pi^-/\sqrt{2} \end{pmatrix}. \quad (8)$$

As regards the matter fields, the quarks of one generation are embedded into the multiplets[37, 40]:

$$\begin{aligned} Q_L^i &= \begin{pmatrix} \chi_{u_L}^i(-+)_5/3 & q_{u_L}^i(++)_{2/3} \\ \chi_{d_L}^i(-+)_2/3 & q_{d_L}^i(++)_{-1/3} \end{pmatrix}, \quad Q_{u_R}^i = u_R^i(++)_{2/3} \\ \tilde{Q}_{d_R}^i &= \begin{pmatrix} X_R^i(-+)_5/3 \\ U_R^i(-+)_2/3 \\ D_R^i(-+)_-1/3 \end{pmatrix}, \quad Q_{d_R}^i = \begin{pmatrix} \tilde{X}_R^i(-+)_5/3 \\ \tilde{U}_R^i(-+)_2/3 \\ d_R^i(++)_{-1/3} \end{pmatrix}, \end{aligned} \quad (9)$$

and the corresponding states of opposite chirality are given by

$$\begin{aligned} Q_R^i &= \begin{pmatrix} \chi_{u_R}^i(+-)_5/3 & q_{u_R}^i(--)_2/3 \\ \chi_{d_R}^i(+-)_2/3 & q_{d_R}^i(--)_-1/3 \end{pmatrix}, \quad Q_{u_L}^i = u_L^i(--)_2/3 \\ \tilde{Q}_{d_L}^i &= \begin{pmatrix} X_L^i(+-)_5/3 \\ U_L^i(+-)_2/3 \\ D_L^i(+-)_-1/3 \end{pmatrix}, \quad Q_{d_L}^i = \begin{pmatrix} \tilde{X}_L^i(+-)_5/3 \\ \tilde{U}_L^i(+-)_2/3 \\ d_L^i(--)_-1/3 \end{pmatrix}. \end{aligned} \quad (10)$$

Here,  $i = 1, 2, 3$  denotes the index of generation, the  $U(1)_X$  charges are all assigned as

$$Y_{Q^i} = Y_{u^i} = Y_{Q_d^i} = \frac{2}{3}. \quad (11)$$

In order to give the kinetic terms of triplets, we redefine the quarks in triplet as

$$\begin{aligned} \tilde{T}_{Q_R}^i &= \begin{pmatrix} \frac{1}{\sqrt{2}}(X_R^i + D_R^i) \\ \frac{i}{\sqrt{2}}(X_R^i - D_R^i) \\ U_R^i \end{pmatrix}, \quad T_{Q_R}^i = \begin{pmatrix} \frac{1}{\sqrt{2}}(\tilde{X}_R^i + d_R^i) \\ \frac{i}{\sqrt{2}}(\tilde{X}_R^i - d_R^i) \\ \tilde{U}_R^i \end{pmatrix}, \\ \tilde{T}_{Q_L}^i &= \begin{pmatrix} \frac{1}{\sqrt{2}}(X_L^i + D_L^i) \\ \frac{i}{\sqrt{2}}(X_L^i - D_L^i) \\ U_L^i \end{pmatrix}, \quad T_{Q_L}^i = \begin{pmatrix} \frac{1}{\sqrt{2}}(\tilde{X}_L^i + d_L^i) \\ \frac{i}{\sqrt{2}}(\tilde{X}_L^i - d_L^i) \\ \tilde{U}_L^i \end{pmatrix}. \end{aligned} \quad (12)$$

the Lagrangian we will use is written as

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_H + \mathcal{L}_Q + \mathcal{L}_Y^Q. \quad (13)$$

where

$$\mathcal{L}_{gauge} = \frac{\sqrt{\mathcal{G}}}{r} \mathcal{G}^{KM} \mathcal{G}^{LN} \left( -\frac{1}{4} W_{KL}^i W_{MN}^i - \frac{1}{4} \tilde{W}_{KL}^i \tilde{W}_{MN}^i - \frac{1}{4} \tilde{B}_{KL} \tilde{B}_{MN} - \frac{1}{4} G_{KL}^a G_{MN}^a \right) \quad (14)$$

is the Lagrangian for gauge sector, and the corresponding Lagrangian for Higgs field is written as

$$\mathcal{L}_H = \text{Tr} \left[ \left( D_\mu \Phi(x) \right)^\dagger \left( D^\mu \Phi(x) \right) \right] - \mu^2 \text{Tr} \left( \Phi^\dagger(x) \Phi(x) \right) + \frac{\lambda}{2} \left[ \text{Tr} \left( \Phi^\dagger(x) \Phi(x) \right) \right]^2 \quad (15)$$

with

$$D_M H = \partial_M H + \frac{i}{2} g_5 \left( \sum_{a=1}^3 W_{L,M}^a \sigma^a \right) H + \frac{i}{2} g_5 H \left( \sum_{a=1}^3 W_{R,M}^a \sigma^a \right)^T. \quad (16)$$

The Lagrangian for kinetic terms of quarks can be written as

$$\begin{aligned} \mathcal{L}_Q = & \frac{\sqrt{\mathcal{G}}}{2r} \sum_{i=1}^3 \left\{ (\bar{Q}^i)_{a_1 a_2} i E_A^M \gamma^A \left[ \left( \frac{1}{2} (\partial_M - \overleftarrow{\partial}_M) + i g_{5s} T^a G_M^a + i g_{5X} Y_{Q^i} \tilde{B}_M \right) \delta_{a_1 b_1} \delta_{a_2 b_2} \right. \right. \\ & + i g_5 \left( \frac{\sigma^{c_1}}{2} \right)_{a_1 b_1} W_{L,M}^{c_1} \delta_{a_2 b_2} + i g_5 \left( \frac{\sigma^{c_2}}{2} \right)_{a_2 b_2} W_{R,M}^{c_2} \delta_{a_1 b_1} \left. \right] (Q^i)_{b_1 b_2} \\ & + (\bar{Q}^i)_{a_1 a_2} \left[ i E_A^M \gamma^A \omega_M - \text{sgn}(\phi) k(c_B)_{ij} \right] (Q^j)_{a_1 a_2} \\ & + \bar{u}^i \left[ i E_A^M \gamma^A \left( \frac{1}{2} (\partial_M - \overleftarrow{\partial}_M) + i g_{5s} T^a G_M^a + i g_{5X} Y_{u^i} \tilde{B}_M \right) \delta_{ij} \right. \\ & + i E_A^M \gamma^A \omega_M - \text{sgn}(\phi) k(c_S)_{ij} \left. \right] u^j \\ & + (\bar{T}_Q^i)_{a_1} i E_A^M \gamma^A \left[ \left( \frac{1}{2} (\partial_M - \overleftarrow{\partial}_M) + i g_{5s} T^a G_M^a + i g_{5X} Y_{Q_d^i} \tilde{B}_M \right) \delta_{a_1 b_1} \right. \\ & + g_5 \varepsilon_{a_1 b_1 c_1} W_{L,M}^{c_1} \left. \right] (\tilde{T}_Q^i)_{b_1} + (\bar{T}_Q^i)_{a_1} \left[ i E_A^M \gamma^A \omega_M - \text{sgn}(\phi) (\eta_3)_{ij} \right] (\tilde{T}_Q^j)_{a_1} \\ & + (\bar{T}_Q^i)_{a_1} i E_A^M \gamma^A \left[ \left( \frac{1}{2} (\partial_M - \overleftarrow{\partial}_M) + i g_{5s} T^a G_M^a + i g_{5X} Y_{Q_d^i} \tilde{B}_M \right) \delta_{a_1 b_1} \right. \\ & + g_5 \varepsilon_{a_1 b_1 c_1} W_{R,M}^{c_1} \left. \right] (\tilde{T}_Q^i)_{b_1} + (\bar{T}_Q^i)_{a_1} \left[ i E_A^M \gamma^A \omega_M - \text{sgn}(\phi) k(c_T)_{ij} \right] (\tilde{T}_Q^j)_{a_1} + h.c. \left. \right\} \quad (17) \end{aligned}$$

with  $\gamma^A = (\gamma^\mu, -i\gamma^5)$ , the inverse vielbein  $E_B^A = \text{diag}(e^{\sigma(\phi)}, e^{\sigma(\phi)}, e^{\sigma(\phi)}, e^{\sigma(\phi)}, \frac{1}{r})$ , and the spin connection  $\omega_A = (\text{sgn}(\phi) \frac{i}{2} k e^{-\sigma(\phi)} \gamma_\mu \gamma^5, 0)$ . Generally, three bulk mass matrices  $c_B, c_S, c_T$  are arbitrarily hermitian  $3 \times 3$  matrices. Furthermore, we also assume that  $c_B, c_S, c_T$  are real and diagonal, i.e. each of them is described by three real parameters. This can always be obtained through some appropriate field redefinitions.

At last,

$$\begin{aligned} \mathcal{L}_Y^Q = e^{kr\pi/2} \sqrt{-\mathcal{G}_{IR}} \sum_{i,j=1}^3 \left\{ \sqrt{2} \lambda_{ij}^u \overline{Q}_{a\alpha}^i H_{a\alpha} u^j - 2 \lambda_{ij}^d \left[ \overline{Q}_{a\alpha}^i (\tau^c)_{ab} (\tilde{T}_d^j)_c H_{b\alpha} \right. \right. \\ \left. \left. + \overline{Q}_{a\alpha}^i (\tau^c)_{\alpha\beta} (T_d^j)_c H_{a\beta} \right] + h.c. \right\} , \end{aligned} \quad (18)$$

is the Lagrangian for Yukawa couplings between quarks and Higgs field. Here the metric on IR brane  $\mathcal{G}_{IR}^{\mu\nu} = e^{kr\pi/2} \eta^{\mu\nu}$ .

For convenience in our analysis, we define the gauge couplings in 4D which are related to the 5D gauge couplings via

$$\begin{aligned} g &= \frac{g_5}{\sqrt{2\pi r}} , \\ g_X &= \frac{g_{5X}}{\sqrt{2\pi r}} . \end{aligned} \quad (19)$$

Correspondingly, the constant of electromagnetic coupling and Weinberg angle in 4D are given through

$$\begin{aligned} e &= \frac{gg_X}{\sqrt{g^2 + 2g_X^2}} , \\ \sin \theta_w &= \frac{g_X}{\sqrt{g^2 + 2g_X^2}} . \end{aligned} \quad (20)$$

In terms of the Weinberg angle  $\theta_w$  and the constant of electromagnetic coupling  $e$ , the gauge couplings in Eq.(19) are written as

$$\begin{aligned} g &= \frac{e}{\sin \theta_w} , \\ g_X &= \frac{e}{\sqrt{1 - 2 \sin^2 \theta_w}} . \end{aligned} \quad (21)$$

To discuss the phenomenology at EW scale, we write the KK decompositions of 5D gauge



fields in our notations as

$$\begin{aligned}
A_\mu(x, \phi) &= \frac{1}{\sqrt{r}} \sum_{n=0}^{\infty} A_\mu^{(n)}(x) \chi_{(++)}^A(y_{(++)}^{A(n)}, t), \\
Z_\mu(x, \phi) &= \frac{1}{\sqrt{r}} \sum_{n=0}^{\infty} Z_\mu^{(n)}(x) \chi_{(++)}^Z(y_{(++)}^{Z(n)}, t), \\
Z_{X,\mu}(x, \phi) &= \frac{1}{\sqrt{r}} \sum_{n=1}^{\infty} Z_{X,\mu}^{(n)}(x) \chi_{(-+)}^{Z_X}(y_{(-+)}^{Z_X(n)}, t), \\
W_{L,\mu}^\pm(x, \phi) &= \frac{1}{\sqrt{r}} \sum_{n=0}^{\infty} W_{L,\mu}^{\pm(n)}(x) \chi_{(++)}^{W_L}(y_{(++)}^{W_L(n)}, t), \\
W_{R,\mu}^\pm(x, \phi) &= \frac{1}{\sqrt{r}} \sum_{n=1}^{\infty} W_{R,\mu}^{\pm(n)}(x) \chi_{(-+)}^{W_R}(y_{(-+)}^{W_R(n)}, t), \\
G_\mu^a(x, \phi) &= \frac{1}{\sqrt{r}} \sum_{n=0}^{\infty} G_\mu^{a(n)}(x) \chi_{(++)}^g(y_{(++)}^{g(n)}, t).
\end{aligned} \tag{22}$$

Where  $y_{(++)}^{G(n)}$  ( $n = 0, 1, \dots, \infty$ ),  $G = A, Z, W_L^\pm, g$  denote the roots of equation  $z^2 R_{(++)}^{G,\epsilon}(z) \equiv 0$  with

$$R_{(++)}^{G,\epsilon}(z) = Y_0(z)J_0(z\epsilon) - J_0(z)Y_0(z\epsilon). \tag{23}$$

and  $y_{(-+)}^{G(n)}$  ( $n = 1, 2, \dots, \infty$ ),  $G = Z_X, W_R^\pm$  denote the roots of equation  $R_{(-+)}^{G,\epsilon}(z) \equiv 0$  with

$$R_{(-+)}^{G,\epsilon}(z) = Y_0(z)J_1(z\epsilon) - J_0(z)Y_1(z\epsilon). \tag{24}$$

Similarly, the KK decompositions of 5D quark fields are written as

$$\begin{aligned}
\chi_{u_L}^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \chi_{u_L}^{i,(n)}(x) f_{(-+)}^{L, c_B^i}(y_{(\mp\pm)}^{c_B^i(n)}, t), \quad \chi_{d_L}^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \chi_{d_L}^{i,(n)}(x) f_{(-+)}^{L, c_B^i}(y_{(\mp\pm)}^{c_B^i(n)}, t), \\
q_{u_L}^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n q_{u_L}^{i,(n)}(x) f_{(++)}^{L, c_B^i}(y_{(\pm\pm)}^{c_B^i(n)}, t), \quad q_{d_L}^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n q_{d_L}^{i,(n)}(x) f_{(++)}^{L, c_B^i}(y_{(\pm\pm)}^{c_B^i(n)}, t), \\
u_R^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n u_R^{i,(n)}(x) f_{(++)}^{R, c_S^i}(y_{(\mp\mp)}^{c_S^i(n)}, t), \quad X_R^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n X_R^{i,(n)}(x) f_{(-+)}^{R, c_T^i}(y_{(\pm\mp)}^{c_T^i(n)}, t), \\
U_R^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n U_R^{i,(n)}(x) f_{(-+)}^{R, c_T^i}(y_{(\pm\mp)}^{c_T^i(n)}, t), \quad D_R^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n D_R^{i,(n)}(x) f_{(-+)}^{R, c_T^i}(y_{(\pm\mp)}^{c_T^i(n)}, t), \\
\tilde{X}_R^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \tilde{X}_R^{i,(n)}(x) f_{(-+)}^{R, c_T^i}(y_{(\pm\mp)}^{c_T^i(n)}, t), \quad \tilde{U}_R^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \tilde{U}_R^{i,(n)}(x) f_{(-+)}^{R, c_T^i}(y_{(\pm\mp)}^{c_T^i(n)}, t), \\
d_R^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n d_R^{i,(n)}(x) f_{(++)}^{R, c_T^i}(y_{(\mp\mp)}^{c_T^i(n)}, t), \quad X_L^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n X_L^{i,(n)}(x) f_{(+ -)}^{L, c_T^i}(y_{(\pm\mp)}^{c_T^i(n)}, t), \\
U_L^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n U_L^{i,(n)}(x) f_{(+ -)}^{L, c_T^i}(y_{(\pm\mp)}^{c_T^i(n)}, t), \quad D_L^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n D_L^{i,(n)}(x) f_{(+ -)}^{L, c_T^i}(y_{(\pm\mp)}^{c_T^i(n)}, t), \\
\tilde{X}_L^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \tilde{X}_L^{i,(n)}(x) f_{(+ -)}^{L, c_T^i}(y_{(\pm\mp)}^{c_T^i(n)}, t), \quad \tilde{U}_L^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \tilde{U}_L^{i,(n)}(x) f_{(+ -)}^{L, c_T^i}(y_{(\pm\mp)}^{c_T^i(n)}, t), \\
d_L^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n d_L^{i,(n)}(x) f_{(--)}^{L, c_T^i}(y_{(\mp\mp)}^{c_T^i(n)}, t), \quad u_L^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n u_L^{i,(n)}(x) f_{(--)}^{L, c_S^i}(y_{(\mp\mp)}^{c_S^i(n)}, t), \\
\chi_{u_R}^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \chi_{u_R}^{i,(n)}(x) f_{(+ -)}^{R, c_B^i}(y_{(\mp\pm)}^{c_B^i(n)}, t), \quad \chi_{d_R}^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \chi_{d_R}^{i,(n)}(x) f_{(+ -)}^{R, c_B^i}(y_{(\mp\pm)}^{c_B^i(n)}, t), \\
q_{u_R}^i(x, \phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n q_{u_R}^{i,(n)}(x) f_{(--)}^{R, c_B^i}(y_{(\pm\pm)}^{c_B^i(n)}, t), \quad q_{d_R}^i(x, \phi) = \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n q_{d_R}^{i,(n)}(x) f_{(--)}^{R, c_B^i}(y_{(\pm\pm)}^{c_B^i(n)}, t).
\end{aligned} \tag{25}$$

In Eq.(25), the eigenvalues  $y_{(\pm\pm)}^{c(n)}$  ( $n \geq 1$ ) satisfy the equation  $R_{(\pm\pm)}^{c,\epsilon}(z) \equiv 0$ ,  $y_{(\mp\mp)}^{c(n)}$  ( $n \geq 1$ ) satisfy the equation  $R_{(\mp\mp)}^{c,\epsilon}(z) \equiv 0$ , and the eigenvalues  $y_{(\pm\mp)}^{c(n)}$  ( $n \geq 1$ ) satisfy the equation  $R_{(\pm\mp)}^{c,\epsilon}(z) \equiv 0$ , respectively. Here, the concrete

expressions of  $R_{(\pm\pm)}^{c,\epsilon}(z)$ ,  $R_{(\pm\mp)}^{c,\epsilon}(z)$ ,  $R_{(\mp\pm)}^{c,\epsilon}(z)$ ,  $R_{(\mp\mp)}^{c,\epsilon}(z)$  are

$$\begin{aligned}
R_{(\pm\pm)}^{c,\epsilon}(z) &= \begin{cases} Y_N(z)J_N(z\epsilon) - J_N(z)Y_N(z\epsilon), & c = N + \frac{1}{2} \\ J_{-c+\frac{1}{2}}(z)J_{c-\frac{1}{2}}(z\epsilon) - J_{c-\frac{1}{2}}(z)J_{-c+\frac{1}{2}}(z\epsilon), & c \neq N + \frac{1}{2} \end{cases}, \\
R_{(\pm\mp)}^{c,\epsilon}(z) &= \begin{cases} J_{N+1}(z)Y_N(z\epsilon) - Y_{N+1}(z)J_N(z\epsilon), & c = N + \frac{1}{2} \\ J_{c+\frac{1}{2}}(z)J_{-c+\frac{1}{2}}(z\epsilon) + J_{-c-\frac{1}{2}}(z)J_{c-\frac{1}{2}}(z\epsilon), & c \neq N + \frac{1}{2} \end{cases}, \\
R_{(\mp\pm)}^{c,\epsilon}(z) &= \begin{cases} Y_N(z)J_{N+1}(z\epsilon) - J_N(z)Y_{N+1}(z\epsilon), & c = N + \frac{1}{2} \\ J_{-c+\frac{1}{2}}(z)J_{c+\frac{1}{2}}(z\epsilon) + J_{c-\frac{1}{2}}(z)J_{-c-\frac{1}{2}}(z\epsilon), & c \neq N + \frac{1}{2} \end{cases}, \\
R_{(\mp\mp)}^{c,\epsilon}(z) &= \begin{cases} J_{N+1}(z)Y_{N+1}(z\epsilon) - Y_{N+1}(z)J_{N+1}(z\epsilon), & c = N + \frac{1}{2} \\ J_{c+\frac{1}{2}}(z)J_{-c-\frac{1}{2}}(z\epsilon) - J_{-c-\frac{1}{2}}(z)J_{c+\frac{1}{2}}(z\epsilon), & c \neq N + \frac{1}{2} \end{cases}. \quad (26)
\end{aligned}$$

Since the radiative corrections from all virtual KK modes to the physics quantities at electroweak scale should be summed over in principle in order to obtain the theoretical predictions in extensions of the SM with a warped or universal extra dimension. In the Ref.[37], the author verify some lemmas on the eigenvalues of KK modes, and sum over the infinite series of KK modes using the residue theorem. Here we list some useful results:

For gauge fields with  $(++)$  BCs:

$$\begin{aligned}
\Sigma_{(++)}^G(t, t') &= \sum_{n=1}^{\infty} \frac{[\chi_{(++)}^G(y_{(++)}^{G(n)}, t)][\chi_{(++)}^G(y_{(++)}^{G(n)}, t')]}{[y_{(++)}^{G(n)}]^2} \\
&= \frac{1}{8\pi} \left\{ t^2(2\ln t - 1) + t'^2(2\ln t' - 1) \right. \\
&\quad \left. - 2\ln \epsilon [t^2\theta(t' - t) + t'^2\theta(t - t')] - \frac{1 - \epsilon^2}{\ln \epsilon} \right\}, \quad (27)
\end{aligned}$$

which is coincided with Eq.(34) exactly in Ref.[26].

Gauge fields with  $(-+)$  BCs

$$\begin{aligned}
\Sigma_{(-+)}^G(t, t') &= \sum_{n=1}^{\infty} \frac{\left[ \chi_{(-+)}^G(y_{(-+)}^{G(n)}, t) \right] \left[ \chi_{(-+)}^G(y_{(-+)}^{G(n)}, t') \right]}{\left[ y_{(-+)}^{G(n)} \right]^2} \\
&= \frac{1}{4\pi} \left( \frac{-\epsilon \ln \epsilon}{1 - 4\epsilon^2 + 3\epsilon^4 - 4\epsilon^4 \ln \epsilon} \right)^{1/2} \left\{ \theta(t - t') \sqrt{t'} (t'^2 - \epsilon^2) \right. \\
&\quad \times \left[ 1 - \frac{\epsilon^2}{4} + 2\epsilon^2 \ln \epsilon - \frac{1 - 9\epsilon^4 + 8\epsilon^6 + 6\epsilon^2 \ln \epsilon - 12\epsilon^4 \ln \epsilon - 6\epsilon^6 \ln \epsilon}{6(1 - 4\epsilon^2 + 3\epsilon^4 - 4\epsilon^4 \ln \epsilon)} \right. \\
&\quad \left. \left. - \frac{\epsilon^2 t'^2 (\ln t' - \ln \epsilon)}{t'^2 - \epsilon^2} + \frac{t'^2}{4} - \frac{t^2}{2} (1 - 2 \ln t) + \frac{1 - \epsilon^2}{2 \ln \epsilon} \right] + (t \leftrightarrow t') \right\}. \quad (28)
\end{aligned}$$

For the left-handed fields with  $(++)$  BCs

$$\begin{aligned}
\Sigma_{(\pm\pm)}^{L,c}(t, t') &= \sum_{n=1}^{\infty} \frac{\left[ f_{(++)}^{L,c}(y_{(\pm\pm)}^{c(n)}, t) \right] \left[ f_{(++)}^{L,c}(y_{(\pm\pm)}^{c(n)}, t') \right]}{\left[ y_{(\pm\pm)}^{c(n)} \right]^2} \\
&= -\frac{(1 - 2c)\epsilon \ln \epsilon (tt')^{-c}}{(1 - \epsilon^{1-2c})} \frac{1}{4\pi} \left\{ \frac{2(1 - 2c)(1 - \epsilon^{3-2c})}{(3 - 2c)(1 + 2c)(1 - \epsilon^{1-2c})} + \frac{t^2 + t'^2}{1 - 2c} \right. \\
&\quad - \frac{2}{(1 - 2c)(1 + 2c)} \left[ \theta(t' - t) \left( t'^{1+2c} + \epsilon^{1-2c} t^{1+2c} \right) \right. \\
&\quad \left. \left. + \theta(t - t') \left( t^{1+2c} + \epsilon^{1-2c} t'^{1+2c} \right) \right] \right\}. \quad (29)
\end{aligned}$$

When the left-handed fermions satisfy  $(--)$ BCs,

$$\begin{aligned}
\Sigma_{(\mp\mp)}^{L,c}(t,t') &= \sum_{n=1}^{\infty} \frac{[f_{(-)}^{L,c}(y_{(\mp\mp)}^{c(n)},t)][f_{(-)}^{L,c}(y_{(\mp\mp)}^{c(n)},t')]}{[y_{(\mp\mp)}^{c(n)}]^2} \\
&= -\frac{(1-2c)(3+2c)\epsilon^{3/2+c}\ln\epsilon(tt')^{1+c}}{[\zeta_{(uv)}^{(-)}(c,\epsilon)\zeta_{(ir)}^{(-)}(c,\epsilon)]^{1/2}} \frac{1}{4\pi} \left\{ \theta(t-t')(1-t^{-1-2c})(t'^{-1-2c}-\epsilon^{-1-2c}) \right. \\
&\quad \times \left[ \frac{1}{1-\epsilon^{1+2c}} \left( \frac{1-\epsilon^{3+2c}}{3+2c} + \frac{\epsilon^2-\epsilon^{1+2c}}{1-2c} \right) + \frac{\epsilon^{3+2c}+\epsilon^{1-2c}-2\epsilon^2-\epsilon^4}{[\zeta_{(ir)}^{(-)}(c,\epsilon)]} \right. \\
&\quad - \frac{3(1+2c)^2}{(3-2c)(5+2c)[\zeta_{(ir)}^{(-)}(c,\epsilon)]} + \frac{(1-2c)\epsilon^{5+2c}}{(5+2c)[\zeta_{(ir)}^{(-)}(c,\epsilon)]} + \frac{(3+2c)\epsilon^{3-2c}}{(3-2c)[\zeta_{(ir)}^{(-)}(c,\epsilon)]} \\
&\quad - \frac{1+2\epsilon^2-\epsilon^{1-2c}-\epsilon^{3+2c}}{[\zeta_{(uv)}^{(-)}(c,\epsilon)]} - \frac{3(1+2c)^2\epsilon^4}{(3-2c)(5+2c)[\zeta_{(uv)}^{(-)}(c,\epsilon)]} + \frac{(1-2c)\epsilon^{-1-2c}}{(5+2c)[\zeta_{(uv)}^{(-)}(c,\epsilon)]} \\
&\quad + \frac{(3+2c)\epsilon^{1+2c}}{(3-2c)[\zeta_{(uv)}^{(-)}(c,\epsilon)]} + \frac{1-t^{1-2c}}{(1-2c)(1-t^{-1-2c})} + \frac{t^2-t^{-1-2c}}{(3+2c)(1-t^{-1-2c})} \\
&\quad \left. + \frac{t'^{1-2c}-\epsilon^{1-2c}}{(1-2c)(t'^{-1-2c}-\epsilon^{-1-2c})} + \frac{\epsilon^2 t'^{-1-2c}-\epsilon^{-1-2c} t'^2}{(3+2c)(t'^{-1-2c}-\epsilon^{-1-2c})} \right] + (t \leftrightarrow t') \Big\}. \tag{30}
\end{aligned}$$

For the left-handed fields with  $(+-)$ BCs

$$\begin{aligned}
\Sigma_{(\pm\mp)}^{L,c}(t,t') &= \sum_{n=1}^{\infty} \frac{[f_{(+)}^{L,c}(y_{(\pm\mp)}^{c(n)},t)][f_{(+)}^{L,c}(y_{(\pm\mp)}^{c(n)},t')]}{[y_{(\pm\mp)}^{c(n)}]^2} \\
&= -\frac{\epsilon\ln\epsilon}{8\pi} \left( \frac{(1-2c)^2(3+2c)}{(1-\epsilon^{1-2c})[\zeta_{(ir)}^{(-)}(c,\epsilon)]} \right)^{1/2} \left\{ \theta(t-t')t^{1+c}t'^{-c}|1-t^{-1-2c}| \right. \\
&\quad \times \left[ \frac{2}{1-2c} + \frac{2\epsilon^2}{1+2c} - \frac{4\epsilon^{1-2c}}{(1-2c)(1+2c)} + \frac{\epsilon^{3+2c}+\epsilon^{1-2c}-2\epsilon^2-\epsilon^4}{[\zeta_{(ir)}^{(-)}(c,\epsilon)]} \right. \\
&\quad - \frac{3(1+2c)^2}{(3-2c)(5+2c)[\zeta_{(ir)}^{(-)}(c,\epsilon)]} + \frac{(1-2c)\epsilon^{5+2c}}{(5+2c)[\zeta_{(ir)}^{(-)}(c,\epsilon)]} + \frac{(3+2c)\epsilon^{3-2c}}{(3-2c)[\zeta_{(ir)}^{(-)}(c,\epsilon)]} \\
&\quad + \frac{(3-2c)\epsilon^{1-2c}-2(1-2c)\epsilon^{3-2c}}{(3-2c)(1+2c)(1-\epsilon^{1-2c})} + \frac{1-t^{1-2c}}{(1-2c)(1-t^{-1-2c})} + \frac{t^2-t^{-1-2c}}{(3+2c)(1-t^{-1-2c})} \\
&\quad \left. + \frac{t'^2}{1-2c} - \frac{2\epsilon^{1-2c}t'^{1+2c}}{(1-2c)(1+2c)} \right] + (t \leftrightarrow t') \Big\}. \tag{31}
\end{aligned}$$

Left-handed fermions with  $(-+)$ BCs

$$\begin{aligned}
\Sigma_{(\mp\pm)}^{L,c}(t,t') &= \sum_{n=1}^{\infty} \frac{\left[ f_{(-+)}^{L,c}(y_{(\mp\pm)}^{c(n)}, t) \right] \left[ f_{(-+)}^{L,c}(y_{(\mp\pm)}^{c(n)}, t') \right]}{\left[ y_{(\mp\pm)}^{c(n)} \right]^2} \\
&= -\frac{\epsilon^{3/2+c} \ln \epsilon}{8\pi} \left( \frac{(1-2c)^2(3+2c)}{(1-\epsilon^{1-2c})[\zeta_{(uv)}^{(-)}(c, \epsilon)]} \right)^{1/2} \left\{ \theta(t-t') t^{-c} t'^{1+c} |t'^{-1-2c} - \epsilon^{-1-2c}| \right. \\
&\quad \times \left[ \frac{2}{1+2c} + \frac{2\epsilon^2}{1-2c} - \frac{4\epsilon^{1+2c}}{(1-2c)(1+2c)} + \frac{2(1-2c) + (1+2c)\epsilon^{3-2c} - (3-2c)\epsilon^2}{(3-2c)(1+2c)(1-\epsilon^{1-2c})} \right. \\
&\quad - \frac{1+2\epsilon^2 - \epsilon^{1-2c} - \epsilon^{3+2c}}{[\zeta_{(uv)}^{(-)}(c, \epsilon)]} - \frac{3(1+2c)^2\epsilon^4}{(3-2c)(5+2c)[\zeta_{(uv)}^{(-)}(c, \epsilon)]} + \frac{(1-2c)\epsilon^{-1-2c}}{(5+2c)[\zeta_{(uv)}^{(-)}(c, \epsilon)]} \\
&\quad + \frac{(3+2c)\epsilon^{1+2c}}{(3-2c)[\zeta_{(uv)}^{(-)}(c, \epsilon)]} + \frac{t^2}{1-2c} - \frac{2t^{1+2c}}{(1-2c)(1+2c)} + \frac{t'^{1-2c} - \epsilon^{1-2c}}{(1-2c)(t'^{-1-2c} - \epsilon^{-1-2c})} \\
&\quad \left. \left. + \frac{\epsilon^2 t'^{-1-2c} - \epsilon^{-1-2c} t'^2}{(3+2c)(t'^{-1-2c} - \epsilon^{-1-2c})} \right] + (t \leftrightarrow t') \right\}. \tag{32}
\end{aligned}$$

with

$$\begin{aligned}
\zeta_{(uv)}^{(-)}(c, \epsilon) &= (3+2c)\epsilon^{1+2c} + (1-2c)\epsilon^{-1-2c} - (1+2c)^2\epsilon^2 - (1-2c)(3+2c), \\
\zeta_{(ir)}^{(-)}(c, \epsilon) &= -(3+2c)\epsilon^{1-2c} - (1-2c)\epsilon^{3+2c} + (1+2c)^2 + (1-2c)(3+2c)\epsilon^2. \tag{33}
\end{aligned}$$

Similarly, for the right-handed fields, one analogously has

$$\begin{aligned}
\Sigma_{(\mp\mp)}^{R,c}(t,t') &= \sum_{n=1}^{\infty} \frac{\left[ f_{(++)}^{R,c}(y_{(\mp\mp)}^{c(n)}, t) \right] \left[ f_{(++)}^{R,c}(y_{(\mp\mp)}^{c(n)}, t') \right]}{\left[ y_{(\mp\mp)}^{c(n)} \right]^2} = \Sigma_{(\pm\pm)}^{L,-c}(t,t'), \\
\Sigma_{(\pm\pm)}^{R,c}(t,t') &= \sum_{n=1}^{\infty} \frac{\left[ f_{(--)}^{R,c}(y_{(\mp\mp)}^{c(n)}, t) \right] \left[ f_{(--)}^{R,c}(y_{(\pm\pm)}^{c(n)}, t') \right]}{\left[ y_{(\pm\pm)}^{c(n)} \right]^2} = \Sigma_{(\mp\mp)}^{L,-c}(t,t'), \\
\Sigma_{(\mp\pm)}^{R,c}(t,t') &= \sum_{n=1}^{\infty} \frac{\left[ f_{(+-)}^{R,c}(y_{(\mp\mp)}^{c(n)}, t) \right] \left[ f_{(+-)}^{R,c}(y_{(\mp\pm)}^{c(n)}, t') \right]}{\left[ y_{(\mp\pm)}^{c(n)} \right]^2} = \Sigma_{(\pm\mp)}^{L,-c}(t,t'), \\
\Sigma_{(\pm\mp)}^{R,c}(t,t') &= \sum_{n=1}^{\infty} \frac{\left[ f_{(-+)}^{R,c}(y_{(\pm\mp)}^{c(n)}, t) \right] \left[ f_{(-+)}^{R,c}(y_{(\pm\mp)}^{c(n)}, t') \right]}{\left[ y_{(\pm\mp)}^{c(n)} \right]^2} = \Sigma_{(\mp\pm)}^{L,-c}(t,t'). \tag{34}
\end{aligned}$$

### III. THE THEORETICAL CALCULATION ON THE $t \rightarrow c\gamma$ AND $t \rightarrow cg$ PROCESSES

In this section, we present one-loop radiative corrections to the rare decay  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  in the extension of the SM with a warped extra dimension and the custodial symmetry.

The infinite dimensional column vectors for quarks in the chirality basis as[40]

$$\begin{aligned}
\Psi_L(5/3) &= \left( \chi_{u_L}^{i(n)}(-+), X_L^{i(n)}(+ -), \tilde{X}_L^{i(n)}(+ -), \dots \right)^T, \\
\Psi_R(5/3) &= \left( \chi_{u_R}^{i(n)}(+ -), X_R^{i(n)}(- +), \tilde{X}_R^{i(n)}(- +), \dots \right)^T, \\
\Psi_L(2/3) &= \left( q_{u_L}^{i(0)}(++) , \dots , q_{u_L}^{i(n)}(++) , U_L^{i(n)}(+ -), \tilde{U}_L^{i(n)}(+ -), \chi_{d_L}^{i(n)}(- +), u_L^{i(n)}(--), \dots \right)^T, \\
\Psi_R(2/3) &= \left( u_R^{i(0)}(++) , \dots , q_{u_R}^{i(n)}(--), U_R^{i(n)}(- +), \tilde{U}_R^{i(n)}(- +), \chi_{d_R}^{i(n)}(+ -), u_R^{i(n)}(++) , \dots \right)^T, \\
\Psi_L(-1/3) &= \left( q_{d_L}^{i(0)}(++) , \dots , q_{d_L}^{i(n)}(++) , D_L^{i(n)}(+ -), d_L^{i(n)}(--), \dots \right)^T, \\
\Psi_R(-1/3) &= \left( d_R^{i(0)}(++) , \dots , q_{d_R}^{i(n)}(--), D_R^{i(n)}(- +), d_R^{i(n)}(++) , \dots \right)^T,
\end{aligned} \tag{35}$$

where the flavor index  $i = 1, 2, 3$  runs over the three quark generations, and  $n = 1, 2, \dots, \infty$  is the index of KK exciting modes, the signs in parentheses denote the BCs satisfied by corresponding fields on UV and IR branes respectively.

We can write the mass eigenstates of charged 5/3, 2/3 and  $-1/3$  quarks as

$$\begin{aligned}
H_{\alpha,L} &= \left[ \mathcal{H}_L^\dagger \Psi_L(5/3) \right]_\alpha, H_{\alpha,R} = \left[ \mathcal{H}_R^\dagger \Psi_R(5/3) \right]_\alpha, \\
U_{\alpha,L} &= \left[ \mathcal{U}_L^\dagger \Psi_L(2/3) \right]_\alpha, U_{\alpha,R} = \left[ \mathcal{U}_R^\dagger \Psi_R(2/3) \right]_\alpha, \\
D_{\alpha,L} &= \left[ \mathcal{D}_L^\dagger \Psi_L(-1/3) \right]_\alpha, D_{\alpha,R} = \left[ \mathcal{D}_R^\dagger \Psi_R(-1/3) \right]_\alpha.
\end{aligned} \tag{36}$$

Here, the charged 2/3 quarks  $U_1, U_2, U_3$  are identified as up-type quarks  $u, c, t$ , and the charged  $-1/3$  quarks  $D_1, D_2, D_3$  are identified as the down-type quarks  $d, s, b$  in the SM, respectively, and  $H_{\alpha,L}, H_{\alpha,R}$  are the charged 5/3 quarks, which not exist in the SM.

Similarly, we could express interaction eigenstates of charged and neutral electroweak

gauge bosons in linear combination of the mass eigenstates as

$$\begin{aligned}
W_L^{(0)\pm} &= \left(\mathcal{Z}_W\right)_{0,0} W^\pm + \sum_{\alpha=1}^{\infty} \left(\mathcal{Z}_W\right)_{0,\alpha} W_{H_\alpha}^\pm, \\
W_L^{(n)\pm} &= \left(\mathcal{Z}_W\right)_{2n-1,0} W^\pm + \sum_{\alpha=1}^{\infty} \left(\mathcal{Z}_W\right)_{2n-1,\alpha} W_{H_\alpha}^\pm, \\
W_R^{(n)\pm} &= \left(\mathcal{Z}_W\right)_{2n,0} W^\pm + \sum_{\alpha=1}^{\infty} \left(\mathcal{Z}_W\right)_{2n,\alpha} W_{H_\alpha}^\pm, \\
Z^{(0)} &= \left(\mathcal{Z}_Z\right)_{0,0} Z + \sum_{\alpha=1}^{\infty} \left(\mathcal{Z}_Z\right)_{0,\alpha} Z_{H_\alpha}, \\
Z^{(n)} &= \left(\mathcal{Z}_Z\right)_{2n-1,0} Z + \sum_{\alpha=1}^{\infty} \left(\mathcal{Z}_Z\right)_{2n-1,\alpha} Z_{H_\alpha}, \\
Z_X^{(n)} &= \left(\mathcal{Z}_Z\right)_{2n,0} Z + \sum_{\alpha=1}^{\infty} \left(\mathcal{Z}_Z\right)_{2n,\alpha} Z_{H_\alpha}, \tag{37}
\end{aligned}$$

in which  $\mathcal{Z}_W$ ,  $\mathcal{Z}_Z$  respectively denote the mixing matrices for charged as well as neutral electroweak gauge bosons, and  $Z, W^\pm$  are identified as the corresponding gauge bosons in the SM.

The relevant Feynman diagrams are draw in Fig.1 when we adopt the background gauge[41]

We could see that the FCNC transitions are mediated by Higgs, the KK excitations of gluon, photo, and neutral electroweak gauge bosons besides the charged electroweak gauge bosons  $W^\pm$  together with their KK partners.

In order to evaluate there diagrams, and sum over the infinite series of KK modes, we divided them into two cases: When all the propagators are SM particles, such as  $Z, W^\pm, H_0, G_0, G^\pm, U_i = u_i, D_i = d_i$  ( $i = 1, 2, 3$ ), we use the general method in [1].

And when the propagator is the KK mode, such as  $Z_{H_\alpha}, W_{H_\alpha}^\pm, \gamma_{(n)}, g_{(n)}, U_{(3+\beta)}, D_{(3+\beta)}, H_{(3+\beta)}$  ( $n, \alpha, \beta = 1, 2, \dots, \infty$ ), since the mass of the KK mode is large then the top quark, we could use the effective Hamilton method[37] and expand the amplitudes to the order  $m_t^2/\Lambda_{KK}^2$  for simplicity.



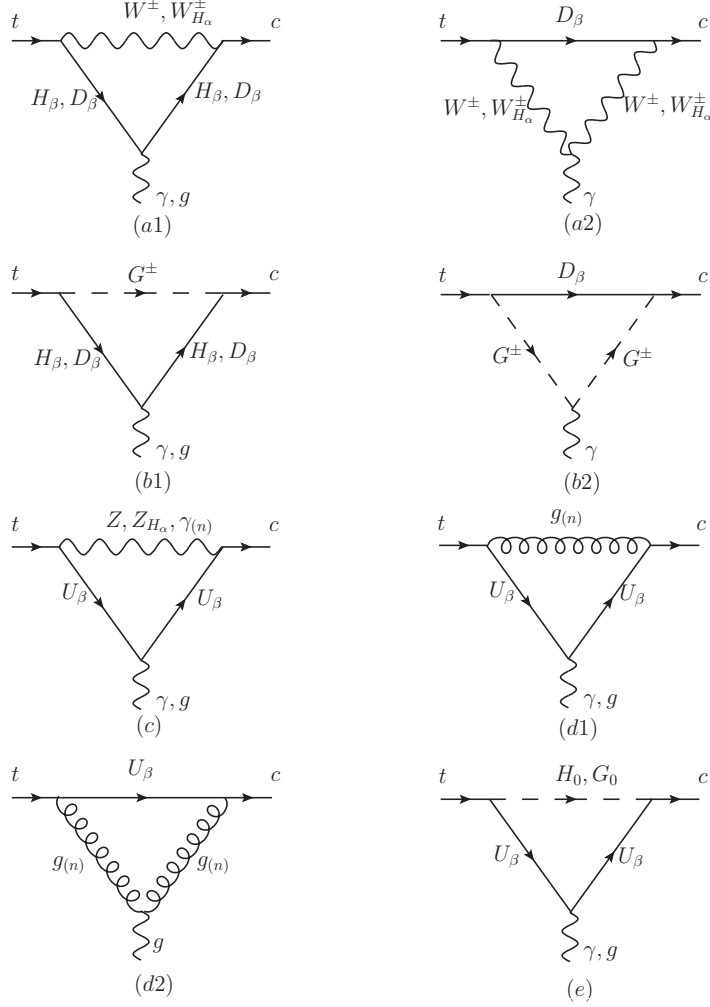


FIG. 1: The Feynman diagrams contributing to the  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  decay in warped extra dimension with custodial symmetry. Where  $Z$ ,  $W^\pm$ ,  $H_0$ ,  $G_0$ ,  $G^\pm$ ,  $U_i = u_i$ ,  $D_i = d_i$  ( $i = 1, 2, 3$ ) denote the normally neutral and charged gauge bosons, neutral Higgs, neutral and charged Goldstones, and three generation up- and down-type quarks,  $Z_{H_\alpha}$ ,  $W_{H_\alpha}^\pm$ ,  $\gamma_{(n)}$ ,  $g_{(n)}$ ,  $U_{(3+\beta)}$ ,  $D_{(3+\beta)}$ ,  $H_{(3+\beta)}$  ( $n, \alpha, \beta = 1, 2, \dots, \infty$ ) denote those heavy gauge bosons together with charge  $2/3$ ,  $-1/3$  and  $5/3$  quarks, respectively.

### A. The first case

When all the propagators are SM particles, the effective flavor changing current for  $t \rightarrow c\gamma, g$  can be written in conserved form

$$J_\mu^{\gamma,g} = \bar{c}(p)[q^2\gamma_\mu(F_L^{\gamma,g}P_L + F_R^{\gamma,g}P_R) + i\sigma_{\mu\nu}q_\nu(m_cF_{TL}^{\gamma,g}P_L + m_tF_{TR}^{\gamma,g}P_R)]t(p') \quad (38)$$

where  $p'$  is the momentum of the initial top quark and  $p$  is the momentum of the final state charm quark,  $q_\mu = p - p'$ , and we have dropped the  $q_\mu$  term because it does not contribute to the processes.

In the equations below,  $m_i$  ( $i = 1, 2, 3$ ) is the mass of the SM three generation up- or down-type quarks,  $m_W$  and  $m_Z$  are the mass of the SM neutral and charged gauge bosons,  $m_{G^\pm}$ ,  $m_{H_0/G_0}$  are the mass of  $G^\pm$  and  $G_{H_0}, G_{G_0}$  respectively,  $C_{ij}$  are the coefficients of the Lorentz-covariant tensors in the 3-point standard scalar Passarino-Veltman integrals (Eq.(4.7) in Ref. [42]), and it could be calculated by using 'LoopTools'. And the relevant coefficients  $\xi^{L,R}$  and  $\eta^{L,R}$  are the couplings between bosons and quarks, we approximate them to the order  $\mathcal{O}(v^2/\Lambda_{KK}^2)$  and present in Appendix.

### 1. Fig.(a1) and (a2) with SM propagators

In Fig. (a1) and (a2), when one-loop diagrams are composed by the zero mode of charge gauge bosons  $W^\pm$  and charged  $-1/3$  quarks  $d_i$  ( $i = 1, 2, 3$ ),  $F_{TL}^{\gamma,g}$  and  $F_{TR}^{\gamma,g}$  are formulated as

$$\begin{aligned} F_{TL}^{\gamma(a1)} &= \frac{ie^3Q^F}{16\pi^2m_c s_W^2} \left( 2(\xi_{W^\pm}^{R(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} m_i C_1 + (\xi_{W^\pm}^{R(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{R(-1/3)})_{i,t} m_t C_{12} \right. \\ &\quad \left. - (\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} m_c (C_1 - C_{11} + C_{12}) \right) \\ F_{TR}^{\gamma(a1)} &= \frac{ie^3Q^F}{16\pi^2m_t s_W^2} \left( 2(\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{R(-1/3)})_{i,t} m_i C_1 + (\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} m_t C_{12} \right. \\ &\quad \left. - (\xi_{W^\pm}^{R(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{R(-1/3)})_{i,t} m_c (C_1 - C_{11} + C_{12}) \right) \end{aligned} \quad (39)$$

with  $C_{ij} = C_{ij}(p^2, (2p - p')^2, (p - p')^2, m_i^2, m_W^2, m_i^2)$

$$\begin{aligned}
F_{TL}^{\gamma(a2)} &= -\frac{ie^3}{16\pi^2 m_c s_W^2} \left( 2(\xi_{W^\pm}^{R(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} (C_1 + C_2) m_i \right. \\
&\quad + (\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} (C_{11} + C_{12} - C_2) m_c \\
&\quad \left. + (\xi_{W^\pm}^{R(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{R(-1/3)})_{i,t} (-C_1 + C_{12} + C_{22}) m_t \right) \\
F_{TR}^{\gamma(a2)} &= -\frac{ie^3}{16\pi^2 m_t s_W^2} \left( 2(\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{R(-1/3)})_{i,t} (C_1 + C_2) m_i \right. \\
&\quad + (\xi_{W^\pm}^{R(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{R(-1/3)})_{i,t} (C_{11} + C_{12} - C_2) m_c \\
&\quad \left. + (\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} (-C_1 + C_{12} + C_{22}) m_t \right)
\end{aligned} \tag{40}$$

with  $C_{ij} = C_{ij}(p^2, (p-p')^2, p'^2, m_i^2, m_W^2, m_W^2)$

and

$$\begin{aligned}
F_{TL}^{g(a1)} &= \frac{ie^2 g_S T^a}{16\pi^2 m_c s_W^2} \left( 2(\xi_{W^\pm}^{R(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} m_i C_1 \right. \\
&\quad \left. + (\xi_{W^\pm}^{R(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{R(-1/3)})_{i,t} m_t C_{12} - (\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} m_c (C_1 - C_{11} + C_{12}) \right) \\
F_{TR}^{g(a1)} &= \frac{ie^2 g_S T^a}{16\pi^2 m_t s_W^2} \left( 2(\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{R(-1/3)})_{i,t} m_i C_1 \right. \\
&\quad \left. + (\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} m_t C_{12} - (\xi_{W^\pm}^{R(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{R(-1/3)})_{i,t} m_c (C_1 - C_{11} + C_{12}) \right)
\end{aligned} \tag{41}$$

with  $C_{ij} = C_{ij}(p^2, (2p-p')^2, (p-p')^2, m_i^2, m_W^2, m_i^2)$

Using  $\xi^{L,R}$  in appendix, we can approximate the relevant coefficients above as

$$\begin{aligned}
(\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} &= \sum_{i=1}^3 (V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{it}^\dagger + \sum_{i=1}^3 (\Upsilon_{i,1}^{(a)})_{ct} + O\left(\frac{v^4}{\Lambda_{KK}^4}\right), \\
(\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{R(-1/3)})_{i,t} &= \frac{v^2}{2\Lambda_{KK}^2} \sum_{i=1}^3 (V_{CKM}^{(0)})_{ci} (\Delta_{W^\pm}^R)_{it}^\dagger + O\left(\frac{v^4}{\Lambda_{KK}^4}\right), \\
(\xi_{W^\pm}^{R(-1/3)})_{c,i}^\dagger (\xi_{W^\pm}^{L(-1/3)})_{i,t} &= \frac{v^2}{2\Lambda_{KK}^2} \sum_{i=1}^3 (\Delta_{W^\pm}^R)_{ci} (V_{CKM}^{(0)})_{it}^\dagger + O\left(\frac{v^4}{\Lambda_{KK}^4}\right),
\end{aligned} \tag{42}$$

here we define the short-cut notation

$$\begin{aligned}
(\Upsilon_{i,1}^{(a)})_{ct} = & (V_{CKM}^{(0)})_{ci}(V_{CKM}^{(0)}\delta Z_L^d)_{it}^\dagger + (V_{CKM}^{(0)}\delta Z_L^d)_{ci}(V_{CKM}^{(0)})_{it}^\dagger + (V_{CKM}^{(0)})_{ci}(\delta Z_L^{u\dagger}V_{CKM}^{(0)})_{it}^\dagger \\
& + (\delta Z_L^{u\dagger}V_{CKM}^{(0)})_{ci}(V_{CKM}^{(0)})_{it}^\dagger - \frac{v^2}{4\Lambda_{KK}^2}[(V_{CKM}^{(0)})_{ci}(\Delta_{W^\pm}^L)_{it} + (\Delta_{W^\pm}^L)_{ci}(V_{CKM}^{(0)})_{it}^\dagger] .(43)
\end{aligned}$$

where  $\delta Z_L^u, \delta Z_L^d$  are the correct to the mixing matrices, which are presented in the equation (154) and (160) of Ref. [37]. And  $(\xi_{W^\pm}^{L(-1/3)})_{c,i}^\dagger(\xi_{W^\pm}^{L(-1/3)})_{i,t} = \sum_{i=1}^3 (V_{CKM}^{(0)})_{ci}(V_{CKM}^{(0)})_{it}^\dagger$  is just the SM case

## 2. Fig.(b1) and (b2) with SM propagators

The contributions to  $F_{TL}^{\gamma,g}$  and  $F_{TR}^{\gamma,g}$  of diagrams (b1) and (b2) with the zero mode of charge Goldstone and charge  $-1/3$  quarks and charge Higgs  $G^\pm$  are

$$\begin{aligned}
F_{TL}^{\gamma(b1)} = & \frac{ieQ^F}{16\pi^2m_c} \left( (\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} (C_0 + C_1)m_i \right. \\
& \left. + (\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} (C_{12} - C_{11})m_c - (\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} (C_1 + C_{12})m_t \right) \\
F_{TR}^{\gamma(b1)} = & \frac{ieQ^F}{16\pi^2m_t} \left( (\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} (C_0 + C_1)m_i \right. \\
& \left. + (\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} (C_{12} - C_{11})m_c - (\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} (C_1 + C_{12})m_t \right)
\end{aligned} \tag{44}$$

with  $C_{ij} = C_{ij}(p^2, (2p - p')^2, (p - p')^2, m_i^2, m_{G^\pm}^2, m_i^2)$

$$\begin{aligned}
F_{TL}^{\gamma(b2)} = & \frac{ie}{16\pi^2m_c} \left( (\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} (C_0 + C_1 + C_2)m_i \right. \\
& - (\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} (C_1 + C_{11} + C_{12})m_c \\
& \left. - (\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} (C_{12} + C_2 + C_{22})m_t \right) \\
F_{TR}^{\gamma(b2)} = & \frac{ie}{16\pi^2m_t} \left( (\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} (C_0 + C_1 + C_2)m_i \right. \\
& - (\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} (C_1 + C_{11} + C_{12})m_c \\
& \left. - (\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} (C_{12} + C_2 + C_{22})m_t \right)
\end{aligned} \tag{45}$$

with  $C_{ij} = C_{ij}(p^2, (p - p')^2, p'^2, m_i^2, m_{G^\pm}^2, m_{G^\pm}^2)$

and

$$\begin{aligned}
F_{TL}^{g(b1)} &= \frac{ig_S T^a}{16\pi^2 m_c} \left( (\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} (C_0 + C_1) m_i \right. \\
&\quad \left. + (\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} (C_{12} - C_{11}) m_c - (\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} (C_1 + C_{12}) m_t \right) \\
F_{TR}^{g(b1)} &= \frac{ig_S T^a}{16\pi^2 m_t} \left( (\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} (C_0 + C_1) m_i \right. \\
&\quad \left. + (\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} (C_{12} - C_{11}) m_c - (\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} (C_1 + C_{12}) m_t \right)
\end{aligned} \tag{46}$$

with  $C_{ij} = C_{ij}(p^2, (2p - p')^2, (p - p')^2, m_i^2, m_{G^\pm}^2, m_i^2)$

Using  $\eta^{L,R}$  in appendix, the relevant coefficients above can be approximate to the order  $\mathcal{O}(v^2/\Lambda_{KK}^2)$  as

$$\begin{aligned}
(\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} &= \frac{e^2}{2s_w^2} \sum_{i=1}^3 (V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{it}^\dagger \frac{m_t m_c}{m_W^2} + \frac{e^2}{2s_w^2} \sum_{i=1}^3 (\Upsilon_{i,1}^{(b)})_{ct} + O\left(\frac{v^4}{\Lambda_{KK}^4}\right), \\
(\eta_{G^\pm}^{L(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} &= \frac{e^2}{2s_w^2} \sum_{i=1}^3 (V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{it}^\dagger \frac{m_c m_i}{m_W^2} + \frac{e^2}{2s_w^2} \sum_{i=1}^3 (\Upsilon_{i,2}^{(b)})_{ct} + O\left(\frac{v^4}{\Lambda_{KK}^4}\right), \\
(\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{L(-1/3)})_{i,t} &= \frac{e^2}{2s_w^2} \sum_{i=1}^3 (V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{it}^\dagger \frac{m_t m_i}{m_W^2} + \frac{e^2}{2s_w^2} \sum_{i=1}^3 (\Upsilon_{i,3}^{(b)})_{ct} + O\left(\frac{v^4}{\Lambda_{KK}^4}\right), \\
(\eta_{G^\pm}^{R(-1/3)})_{c,i}^\dagger (\eta_{G^\pm}^{R(-1/3)})_{i,t} &= \frac{e^2}{2s_w^2} \sum_{i=1}^3 (V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{it}^\dagger \frac{m_i^2}{m_W^2} + \frac{e^2}{2s_w^2} \sum_{i=1}^3 (\Upsilon_{i,4}^{(b)})_{ct} + O\left(\frac{v^4}{\Lambda_{KK}^4}\right),
\end{aligned} \tag{47}$$

The first term of each equations are the SM case. And the short-cut notation are

$$\begin{aligned}
(\Upsilon_{i,1}^{(b)})_{ct} &= -(V_{CKM}^{(0)})_{ci}(V_{CKM}^{(0)})_{it}^\dagger \left[ \frac{m_t}{m_W^2} (\delta M^u)_{ii} + \frac{m_c}{m_W^2} (\delta M^u)_{ii}^* \right] \\
&\quad - \frac{2\pi m_t m_c}{\Lambda_{KK}^2} (V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{it}^\dagger \{ [\Sigma_{(++)}^G(1,1)] + [\Sigma_{(-+)}^G(1,1)] \} \\
&\quad + \sum_{j=1}^3 \frac{m_t m_c}{m_W^2} (V_{CKM}^{(0)})_{ci} [(\delta Z_L^d)_{ij} + (\delta Z_L^d)_{ij}^\dagger] (V_{CKM}^{(0)})_{jt}^\dagger \\
&\quad + \sum_{j=1}^3 \frac{m_{d_i}^2}{m_W^2} [(V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{ij}^\dagger (\delta Z_R^u)_{it}^\dagger + (\delta Z_R^u)_{cj} (V_{CKM}^{(0)})_{ji} (V_{CKM}^{(0)})_{it}^\dagger] \\
&\quad + \frac{v^2}{4\Lambda_{KK}^2} \left[ \frac{m_c}{m_W} (V_{CKM}^{(0)})_{ci} (\Delta_{G^\pm}^L)_{it} + \frac{m_t}{m_W} (\Delta_{G^\pm}^L)_{ci}^\dagger (V_{CKM}^{(0)})_{it}^\dagger \right], \\
(\Upsilon_{i,2}^{(b)})_{ct} &= -(V_{CKM}^{(0)})_{ci}(V_{CKM}^{(0)})_{it}^\dagger \left[ \frac{m_{d_i}}{m_{W^\pm}^2} (\delta M^u)_{ii} + \frac{m_c}{m_W^2} (\delta M^d)_{ii}^* \right] \\
&\quad - \frac{\pi}{\Lambda_{KK}^2} (m_c m_{d_i} + m_c m_t) (V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{it}^\dagger \{ [\Sigma_{(++)}^G(1,1)] + [\Sigma_{(-+)}^G(1,1)] \} \\
&\quad + \sum_{j=1}^3 \frac{m_c}{m_W^2} (V_{CKM}^{(0)})_{ci} [m_{d_i} (\delta Z_L^d)_{ij}^\dagger (V_{CKM}^{(0)})_{jt}^\dagger + m_{d_j} (V_{CKM}^{(0)})_{ij}^\dagger (\delta Z_R^u)_{jt}^\dagger] \\
&\quad + \sum_{j=1}^3 \left[ \frac{m_{d_i} m_c}{m_W^2} (V_{CKM}^{(0)})_{cj} (\delta Z_L^d)_{ji} + \frac{m_{d_i} m_{u_j}}{m_W^2} (\delta Z_R^u)_{cj} (V_{CKM}^{(0)})_{ji} \right] (V_{CKM}^{(0)})_{it}^\dagger \\
&\quad + \frac{v^2}{4\Lambda_{KK}^2} \left[ \frac{m_c}{m_W} (V_{CKM}^{(0)})_{ci} (\Delta_{G^\pm}^R)_{it} + \frac{m_{d_i}}{m_W} (\Delta_{G^\pm}^L)_{ci}^\dagger (V_{CKM}^{(0)})_{it}^\dagger \right], \\
(\Upsilon_{i,3}^{(b)})_{ct} &= -(V_{CKM}^{(0)})_{ci}(V_{CKM}^{(0)})_{it}^\dagger \left[ \frac{m_{d_i}}{m_W^2} (\delta M^u)_{ii}^* + \frac{m_t}{m_W^2} (\delta M^d)_{ii} \right] \\
&\quad - \frac{\pi}{\Lambda_{KK}^2} (m_t m_{d_i} + m_c m_t) (V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{it}^\dagger \{ [\Sigma_{(++)}^G(1,1)] + [\Sigma_{(-+)}^G(1,1)] \} \\
&\quad + \sum_{j=1}^3 \frac{m_{d_i}}{m_W^2} (V_{CKM}^{(0)})_{ci} [m_t (\delta Z_L^d)_{ij}^\dagger (V_{CKM}^{(0)})_{jt}^\dagger + m_{u_j} (V_{CKM}^{(0)})_{ij}^\dagger (\delta Z_R^u)_{jt}^\dagger] \\
&\quad + \sum_{j=1}^3 \left[ \frac{m_t m_{d_j}}{m_W^2} (V_{CKM}^{(0)})_{cj} (\delta Z_L^d)_{ji} + \frac{m_{d_i} m_t}{m_W^2} (\delta Z_R^u)_{cj} (V_{CKM}^{(0)})_{ji} \right] (V_{CKM}^{(0)})_{it}^\dagger \\
&\quad + \frac{v^2}{4\Lambda_{KK}^2} \left[ \frac{m_{d_i}}{m_W} (V_{CKM}^{(0)})_{ci} (\Delta_{G^\pm}^L)_{it} + \frac{m_t}{m_W} (\Delta_{G^\pm}^R)_{ci}^\dagger (V_{CKM}^{(0)})_{it}^\dagger \right], \\
(\Upsilon_{i,4}^{(b)})_{ct} &= -(V_{CKM}^{(0)})_{ci}(V_{CKM}^{(0)})_{it}^\dagger \left[ \frac{m_{d_i}}{m_W^2} (\delta M^d)_{ii} + \frac{m_{d_i}}{m_W^2} (\delta M^d)_{ii}^* \right] \\
&\quad - \frac{\pi}{\Lambda_{KK}^2} (m_t m_{d_i} + m_c m_{d_i}) (V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{it}^\dagger \{ [\Sigma_{(++)}^G(1,1)] + [\Sigma_{(-+)}^G(1,1)] \} \\
&\quad + \sum_{j=1}^3 \frac{m_{d_i} m_{d_j}}{m_W^2} (V_{CKM}^{(0)})_{ci} [(\delta Z_L^d)_{ij} + (\delta Z_L^d)_{ij}^\dagger] (V_{CKM}^{(0)})_{jt}^\dagger \\
&\quad + \sum_{j=1}^3 \frac{m_{d_i}^2}{m_W^2} [(V_{CKM}^{(0)})_{ci} (V_{CKM}^{(0)})_{ij}^\dagger (\delta Z_R^u)_{it}^\dagger + (\delta Z_R^u)_{cj} (V_{CKM}^{(0)})_{ji} (V_{CKM}^{(0)})_{it}^\dagger]
\end{aligned}$$

where  $\delta Z_L^u, \delta Z_R^u, \delta Z_L^d, \delta Z_R^d$  are presented in the equation (154) and (160) of Ref. [37] too.

### 3. Fig.(c) with SM propagators

Similarly, the contributions from Fig.(c) with the zero mode of neutral gauge bosons  $Z, Z_{H_\alpha}$  and charge 2/3 quarks are

$$\begin{aligned}
F_{TL}^{\gamma(c)} &= \frac{ie^3 Q^F}{32\pi^2 m_c s_W^2 c_W^2} \left( 2(\xi_Z^{R(2/3)})_{c,i}^\dagger (\xi_Z^{L(2/3)})_{i,t} m_i C_1 \right. \\
&\quad \left. + (\xi_Z^{R(2/3)})_{c,i}^\dagger (\xi_Z^{R(2/3)})_{i,t} m_t C_{12} - (\xi_Z^{L(2/3)})_{c,i}^\dagger (\xi_Z^{L(2/3)})_{i,t} m_c (C_1 - C_{11} + C_{12}) \right) \\
F_{TR}^{\gamma(c)} &= \frac{ie^3 Q^F}{32\pi^2 m_t s_W^2 c_W^2} \left( 2(\xi_Z^{L(2/3)})_{c,i}^\dagger (\xi_Z^{R(2/3)})_{i,t} m_i C_1 \right. \\
&\quad \left. + (\xi_Z^{L(2/3)})_{c,i}^\dagger (\xi_Z^{L(2/3)})_{i,t} m_t C_{12} - (\xi_Z^{R(2/3)})_{c,i}^\dagger (\xi_Z^{R(2/3)})_{i,t} m_c (C_1 - C_{11} + C_{12}) \right)
\end{aligned} \tag{49}$$

with  $C_{ij} = C_{ij}(p^2, (2p - p')^2, (p - p')^2, m_i^2, m_Z^2, m_i^2)$

$$\begin{aligned}
F_{TL}^{g(c)} &= \frac{ie^2 g_S T^a}{32\pi^2 m_c s_W^2 c_W^2} \left( 2(\xi_Z^{R(2/3)})_{c,i}^\dagger (\xi_Z^{L(2/3)})_{i,t} m_i C_1 \right. \\
&\quad \left. + (\xi_Z^{R(2/3)})_{c,i}^\dagger (\xi_Z^{R(2/3)})_{i,t} m_t C_{12} - (\xi_Z^{L(2/3)})_{c,i}^\dagger (\xi_Z^{L(2/3)})_{i,t} m_c (C_1 - C_{11} + C_{12}) \right) \\
F_{TR}^{g(c)} &= \frac{ie^4 g_S T^a}{32\pi^2 m_t s_W^2 c_W^2} \left( 2(\xi_Z^{L(2/3)})_{c,i}^\dagger (\xi_Z^{R(2/3)})_{i,t} m_i C_1 \right. \\
&\quad \left. + (\xi_Z^{L(2/3)})_{c,i}^\dagger (\xi_Z^{L(2/3)})_{i,t} m_t C_{12} - (\xi_Z^{R(2/3)})_{c,i}^\dagger (\xi_Z^{R(2/3)})_{i,t} m_c (C_1 - C_{11} + C_{12}) \right)
\end{aligned} \tag{50}$$

with  $C_{ij} = C_{ij}(p^2, (2p - p')^2, (p - p')^2, m_i^2, m_Z^2, m_i^2)$

and there is no contribution to the processes with zero mode of  $\gamma$ .

The relevant coefficients are approximate to the order  $\mathcal{O}(v^2/\Lambda_{KK}^2)$  as

$$\begin{aligned}
(\xi_Z^{L(2/3)})_{c,i}^\dagger (\xi_Z^{L(2/3)})_{i,t} &= \left(\frac{3-4s_w^2}{6s_w c_w}\right)^2 [\delta_{ci}((\delta Z_L^u)_{it}^\dagger + (\delta Z_L^u)_{it}) + ((\delta Z_L^u)_{ci}^\dagger + (\delta Z_L^u)_{ci})\delta_{it} \\
&\quad + \delta_{ci} \frac{v^2}{2\Lambda_{KK}^2} (\Delta_Z^{L(2/3)})_{it} + \frac{v^2}{2\Lambda_{KK}^2} (\Delta_Z^{L(2/3)})_{ci}^\dagger \delta_{it}] + O\left(\frac{v^4}{\Lambda_{KK}^4}\right) \\
(\xi_Z^{L(2/3)})_{c,i}^\dagger (\xi_Z^{R(2/3)})_{i,t} &= -\frac{2}{3} \frac{s_w}{c_w} \left(\frac{3-4s_w^2}{6s_w c_w}\right) [(\delta Z_L^u)_{ci}^\dagger \delta_{it} + (\delta Z_L^u)_{ci} \delta_{it} + \frac{v^2}{2\Lambda_{KK}^2} (\Delta_Z^{L(2/3)})_{ci}^\dagger \delta_{it} \\
&\quad + \delta_{ci} (\delta Z_R^u)_{it}^\dagger + \delta_{ci} (\delta Z_R^u)_{it} + \delta_{ci} \frac{v^2}{2\Lambda_{KK}^2} (\Delta_Z^{R(2/3)})_{it}] + O\left(\frac{v^4}{\Lambda_{KK}^4}\right) \\
(\xi_Z^{R(2/3)})_{c,i}^\dagger (\xi_Z^{R(2/3)})_{i,t} &= \left\{ \left(-\frac{2}{3} \frac{s_w}{c_w}\right)^2 [\delta_{ci}((\delta Z_R^u)_{it}^\dagger + (\delta Z_R^u)_{it}) + ((\delta Z_R^u)_{ci}^\dagger + (\delta Z_R^u)_{ci})\delta_{it} \right. \\
&\quad \left. + \delta_{ci} \frac{v^2}{2\Lambda_{KK}^2} (\Delta_Z^{R(2/3)})_{it} + \frac{v^2}{2\Lambda_{KK}^2} (\Delta_Z^{R(2/3)})_{ci}^\dagger \delta_{it}] + O\left(\frac{v^4}{\Lambda_{KK}^4}\right) \right\} \\
(\xi_Z^{R(2/3)})_{c,i}^\dagger (\xi_Z^{L(2/3)})_{i,t} &= -\frac{2}{3} \frac{s_w}{c_w} \left(\frac{3-4s_w^2}{6s_w c_w}\right) [\delta_{ci} (\delta Z_L^u)_{it}^\dagger + \delta_{ci} (\delta Z_L^u)_{it} + \delta_{ci} \frac{v^2}{2\Lambda_{KK}^2} (\Delta_Z^{L(2/3)})_{it} \\
&\quad + (\delta Z_R^u)_{ci}^\dagger \delta_{it} + (\delta Z_R^u)_{ci} \delta_{it} + \frac{v^2}{2\Lambda_{KK}^2} (\Delta_Z^{R(2/3)})_{ci}^\dagger \delta_{it}] + O\left(\frac{v^4}{\Lambda_{KK}^4}\right)
\end{aligned} \tag{51}$$

#### 4. Fig.(d1) and (d2) with SM propagators

In Fig.(d1)(d2), the relevant coefficients  $(\xi_{g(n)}^{(2/3)})_{c,i}^\dagger (\xi_{g(n)}^{(2/3)})_{i,t}$  are at order  $O(\frac{v^4}{\Lambda_{KK}^4})$ . So we ignore them.

#### 5. Fig.(e) with SM propagators

Finally, the zero mode of neutral Higgs/Goldstone  $H_0, G_0$  and charge 2/3 quarks contribution in Fig.(e) reads

$$\begin{aligned}
F_{TL}^{\gamma(e)} &= \frac{ieQ^F}{16\pi^2 m_c} \left( (\eta_{H_0/G_0}^{L(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{L(2/3)})_{i,t} (C_0 + C_1) m_i \right. \\
&\quad \left. + (\eta_{H_0/G_0}^{R(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{L(2/3)})_{i,t} (C_{12} - C_{11}) m_c - (\eta_{H_0/G_0}^{L(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{R(2/3)})_{i,t} (C_1 + C_{12}) m_t \right) \\
F_{TR}^{\gamma(e)} &= \frac{ieQ^F}{16\pi^2 m_t} \left( (\eta_{H_0/G_0}^{R(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{R(2/3)})_{i,t} (C_0 + C_1) m_i \right. \\
&\quad \left. + (\eta_{H_0/G_0}^{L(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{R(2/3)})_{i,t} (C_{12} - C_{11}) m_c - (\eta_{H_0/G_0}^{R(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{L(2/3)})_{i,t} (C_1 + C_{12}) m_t \right)
\end{aligned} \tag{52}$$



with  $C_{ij} = C_{ij}(p^2, (2p - p')^2, (p - p')^2, m_i^2, m_{H_0/G_0}^2, m_i^2)$

$$\begin{aligned}
F_{TL}^{g(e)} &= \frac{igsT^a}{16\pi^2 m_c} \left( (\eta_{H_0/G_0}^{L(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{L(2/3)})_{i,t} (C_0 + C_1) m_i \right. \\
&\quad \left. + (\eta_{H_0/G_0}^{R(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{L(2/3)})_{i,t} (C_{12} - C_{11}) m_c - (\eta_{H_0/G_0}^{L(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{R(2/3)})_{i,t} (C_1 + C_{12}) m_t \right) \\
F_{TR}^{g(e)} &= \frac{igsT^a}{16\pi^2 m_t} \left( (\eta_{H_0/G_0}^{R(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{R(2/3)})_{i,t} (C_0 + C_1) m_i \right. \\
&\quad \left. + (\eta_{H_0/G_0}^{L(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{R(2/3)})_{i,t} (C_{12} - C_{11}) m_c - (\eta_{H_0/G_0}^{R(2/3)})_{c,i}^\dagger (\eta_{H_0/G_0}^{L(2/3)})_{i,t} (C_1 + C_{12}) m_t \right)
\end{aligned} \tag{53}$$

with  $C_{ij} = C_{ij}(p^2, (2p - p')^2, (p - p')^2, m_i^2, m_{H_0/G_0}^2, m_i^2)$

The relevant coefficients can be expanded according  $\mathcal{O}(v^2/\Lambda_{KK}^2)$  as

$$\begin{aligned}
(\eta_{H_0}^{L(2/3)})^\dagger_{c,i}(\eta_{H_0}^{L(2/3)})_{i,t} &= (\eta_{G_0}^{L(2/3)})^\dagger_{c,i}(\eta_{G_0}^{L(2/3)})_{i,t} = \frac{e^2}{2s_w^2} \left\{ \frac{m_i}{m_W} \delta_{ci} \frac{m_t}{m_W} (\delta Z_R^u)^\dagger_{it} + \frac{m_i}{m_W} \delta_{ci} \frac{m_i}{m_W} (\delta Z_L^u)_{it} \right. \\
&\quad + \frac{v^2}{4\Lambda_{KK}^2} \frac{m_i}{m_W} \delta_{ci} \left[ \frac{m_t}{m_W} (\Delta_{H_0}^{(1)2/3})_{it} + \frac{m_i}{m_W} (\Delta_{H_0}^{(2)2/3})_{it} \right] \\
&\quad + \frac{m_c}{m_W} (\delta Z_R^u)_{ci} \frac{m_t}{m_W} \delta_{it} + \frac{m_i}{m_W} (\delta Z_L^u)^\dagger_{ci} \frac{m_t}{m_W} \delta_{it} \\
&\quad \left. + \frac{v^2}{4\Lambda_{KK}^2} \frac{m_t}{m_W} \delta_{it} \left[ \frac{m_c}{m_W} (\Delta_{H_0}^{(1)2/3})^\dagger_{ci} + \frac{m_i}{m_W} (\Delta_{H_0}^{(2)2/3})^\dagger_{ci} \right] \right\} + O\left(\frac{v^3}{\Lambda_{KK}^3}\right) \\
(\eta_{H_0}^{L(2/3)})^\dagger_{c,i}(\eta_{H_0}^{R(2/3)})_{i,t} &= (\eta_{G_0}^{L(2/3)})^\dagger_{c,i}(\eta_{G_0}^{R(2/3)})_{i,t} = (\eta_{H_0}^{L(2/3)})^\dagger_{c,i}(\eta_{H_0}^{L(2/3)})^\dagger_{i,t} \\
&= \frac{e^2}{2s_w^2} \left\{ \frac{m_i}{m_W} \delta_{ci} \frac{m_i}{m_W} (\delta Z_R^u)_{it} + \frac{m_i}{m_W} \delta_{ci} \frac{m_t}{m_W} (\delta Z_L^u)^\dagger_{it} \right. \\
&\quad + \frac{v^2}{4\Lambda_{KK}^2} \frac{m_i}{m_W} \delta_{ci} \left[ \frac{m_i}{m_W} (\Delta_{H_0}^{(1)2/3})^\dagger_{it} + \frac{m_t}{m_W} (\Delta_{H_0}^{(2)2/3})^\dagger_{it} \right] \\
&\quad + \frac{m_c}{m_W} (\delta Z_R^u)_{ci} \frac{m_t}{m_W} \delta_{it} + \frac{m_i}{m_W} (\delta Z_L^u)^\dagger_{ci} \frac{m_t}{m_W} \delta_{it} \\
&\quad \left. + \frac{v^2}{4\Lambda_{KK}^2} \left[ \frac{m_c}{m_W} (\Delta_{H_0}^{(1)2/3})^\dagger_{ci} + \frac{m_i}{m_W} (\Delta_{H_0}^{(2)2/3})^\dagger_{ci} \right] \frac{m_t}{m_W} \delta_{it} \right\} + O\left(\frac{v^3}{\Lambda_{KK}^3}\right), \\
(\eta_{H_0}^{R(2/3)})^\dagger_{c,i}(\eta_{H_0}^{L(2/3)})_{i,t} &= (\eta_{G_0}^{R(2/3)})^\dagger_{c,i}(\eta_{G_0}^{L(2/3)})_{i,t} = (\eta_{H_0}^{L(2/3)})_{c,i}(\eta_{H_0}^{L(2/3)})_{i,t} \\
&= \frac{e^2}{2s_w^2} \left\{ \frac{m_i}{m_W} \delta_{ci} \frac{m_t}{m_W} (\delta Z_R^u)^\dagger_{it} + \frac{m_i}{m_W} \delta_{ci} \frac{m_i}{m_W} (\delta Z_L^u)_{it} \right. \\
&\quad + \frac{v^2}{4\Lambda_{KK}^2} \frac{m_i}{m_W} \delta_{ci} \left[ \frac{m_t}{m_W} (\Delta_{H_0}^{(1)2/3})_{it} + \frac{m_i}{m_W} (\Delta_{H_0}^{(2)2/3})_{it} \right] \\
&\quad + \frac{m_i}{m_W} (\delta Z_R^u)^\dagger_{ci} \frac{m_t}{m_W} \delta_{it} + \frac{m_c}{m_W} (\delta Z_L^u)_{ci} \frac{m_t}{m_W} \delta_{it} \\
&\quad \left. + \frac{v^2}{4\Lambda_{KK}^2} \frac{m_t}{m_W} \delta_{it} \left[ \frac{m_i}{m_W} (\Delta_{H_0}^{(1)2/3})_{ci} + \frac{m_c}{m_W} (\Delta_{H_0}^{(2)2/3})_{ci} \right] \right\} + O\left(\frac{v^3}{\Lambda_{KK}^3}\right), \\
(\eta_{H_0}^{R(2/3)})^\dagger_{c,i}(\eta_{H_0}^{R(2/3)})_{i,t} &= (\eta_{G_0}^{R(2/3)})^\dagger_{c,i}(\eta_{G_0}^{R(2/3)})_{i,t} = (\eta_{H_0}^{L(2/3)})_{c,i}(\eta_{H_0}^{L(2/3)})^\dagger_{i,t} \\
&= \frac{e^2}{2s_w^2} \left\{ \frac{m_i}{m_W} \delta_{ci} \frac{m_i}{m_W} (\delta Z_R^u)_{it} + \frac{m_i}{m_W} \delta_{ci} \frac{m_t}{m_W} (\delta Z_L^u)^\dagger_{it} \right. \\
&\quad + \frac{v^2}{4\Lambda_{KK}^2} \frac{m_i}{m_W} \delta_{ci} \left[ \frac{m_i}{m_W} (\Delta_{H_0}^{(1)2/3})^\dagger_{it} + \frac{m_t}{m_W} (\Delta_{H_0}^{(2)2/3})^\dagger_{it} \right] \\
&\quad + \frac{m_i}{m_W} (\delta Z_R^u)^\dagger_{ci} \frac{m_t}{m_W} \delta_{it} + \frac{m_c}{m_W} (\delta Z_L^u)_{ci} \frac{m_t}{m_W} \delta_{it} \\
&\quad \left. + \frac{v^2}{4\Lambda_{KK}^2} \frac{m_t}{m_W} \delta_{it} \left[ \frac{m_i}{m_W} (\Delta_{H_0}^{(1)2/3})_{ci} + \frac{m_c}{m_W} (\Delta_{H_0}^{(2)2/3})_{ci} \right] \right\} + O\left(\frac{v^3}{\Lambda_{KK}^3}\right)
\end{aligned} \tag{54}$$

## B. The second case

When the propagator is the KK mode, since the mass of the KK mode is large then the top quark, we expand the amplitudes to the order  $m_t^2/\Lambda_{KK}^2$  for simplicity.

In a conventional form, the effective Hamilton is written as:

$$H_{eff} = \sum_i C_i(\mu) \mathcal{O}_i \quad (55)$$

with the magnetic and chromomagnetic dipole moment operators are defined through [43]

$$\begin{aligned} \mathcal{O}_{\tau\gamma} &= -\frac{i}{2} m_t \bar{c}_\alpha \sigma^{\mu\nu} P_R t_\alpha F_{\mu\nu}, \\ \mathcal{O}_{\tau\gamma} &= -\frac{i}{2} m_c \bar{c}_\alpha \sigma^{\mu\nu} P_L t_\alpha F_{\mu\nu}, \\ \mathcal{O}_{8g} &= -\frac{i}{2} m_t \bar{c}_\alpha \sigma^{\mu\nu} P_R t_\beta G_{\mu\nu}^a, \\ \mathcal{O}_{8g} &= -\frac{i}{2} m_c \bar{c}_\alpha \sigma^{\mu\nu} P_L t_\beta G_{\mu\nu}^a, \end{aligned} \quad (56)$$

where  $F_{\mu\nu}$  and  $G_{\mu\nu}^a$  are the electromagnetic and strong field strength tensors respectively. And in the momentum representation the operators have the same form as in equation (34).

### 1. Fig.(a1)and (a2) with KK mode propagators

In Fig. (a1)and (a2), when one loop diagrams are composed by the KK mode of charged gauge bosons and the KK mode of charged  $-1/3$  quarks, the corrections to the coefficients

at the EW scale  $\mu_{\text{EW}}$  are formulated as

$$\begin{aligned}
C_{7\gamma}^{(a)} &= \frac{ie^3}{16\pi^2\mu_{\text{EW}}^2 s_W^2} \sum_{\beta=1}^{\infty} \left\{ \left( \xi_{W^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \xi_{W^\pm}^{L(-1/3)} \right)_{\beta,t} F_{1,\gamma}^{(a)}(x_{D_\beta}, x_{W^\pm}) \right. \\
&\quad + \frac{m_{D_\beta}}{m_t} \left( \xi_{W^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \xi_{W^\pm}^{R(-1/3)} \right)_{\beta,t} F_{2,\gamma}^{(a)}(x_{D_\beta}, x_{W^\pm}) \\
&\quad + \sum_{\alpha=1}^{\infty} \left( \xi_{W_{H_\alpha}^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \xi_{W_{H_\alpha}^\pm}^{L(-1/3)} \right)_{\beta,t} F_{1,\gamma}^{(a)}(x_{D_\beta}, x_{W_{H_\alpha}^\pm}) \\
&\quad \left. + \frac{m_{D_\beta}}{m_t} \sum_{\alpha=1}^{\infty} \left( \xi_{W_{H_\alpha}^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \xi_{W_{H_\alpha}^\pm}^{R(-1/3)} \right)_{\beta,t} F_{2,\gamma}^{(a)}(x_{D_\beta}, x_{W_{H_\alpha}^\pm}) \right\}, \\
C_{8G}^{(a)} &= \frac{ie^2 g_s T^a}{16\pi^2\mu_{\text{EW}}^2 s_W^2} \sum_{\beta=1}^{\infty} \left\{ \left( \xi_{W^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \xi_{W^\pm}^{L(-1/3)} \right)_{\beta,t} F_{1,g}^{(a)}(x_{D_\beta}, x_{W^\pm}) \right. \\
&\quad + \frac{m_{D_\beta}}{m_t} \left( \xi_{W^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \xi_{W^\pm}^{R(-1/3)} \right)_{\beta,t} F_{2,g}^{(a)}(x_{D_\beta}, x_{W^\pm}) \\
&\quad + \sum_{\alpha=1}^{\infty} \left( \xi_{W_{H_\alpha}^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \xi_{W_{H_\alpha}^\pm}^{L(-1/3)} \right)_{\beta,t} F_{1,g}^{(a)}(x_{D_\beta}, x_{W_{H_\alpha}^\pm}) \\
&\quad \left. + \frac{m_{D_\beta}}{m_t} \sum_{\alpha=1}^{\infty} \left( \xi_{W_{H_\alpha}^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \xi_{W_{H_\alpha}^\pm}^{R(-1/3)} \right)_{\beta,t} F_{2,g}^{(a)}(x_{D_\beta}, x_{W_{H_\alpha}^\pm}) \right\}, \\
\tilde{C}_{7\gamma}^{(a)} &= C_{7\gamma}^{(a)} \left( \xi_{W^\pm}^{L(-1/3)} \leftrightarrow \xi_{W^\pm}^{R(-1/3)}, \xi_{W_{H_\alpha}^\pm}^{L(-1/3)} \leftrightarrow \xi_{W_{H_\alpha}^\pm}^{R(-1/3)} \right), \\
\tilde{C}_{8G}^{(a)} &= C_{8G}^{(a)} \left( \xi_{W^\pm}^{L(-1/3)} \leftrightarrow \xi_{W^\pm}^{R(-1/3)}, \xi_{W_{H_\alpha}^\pm}^{L(-1/3)} \leftrightarrow \xi_{W_{H_\alpha}^\pm}^{R(-1/3)} \right)
\end{aligned} \tag{57}$$

with  $x_i = m_i^2/\mu_{\text{EW}}^2$ , and  $\beta = 1, 2, \dots, \infty$  is the index of all the KK exciting modes and all the three quark generations in each KK exciting modes. And the form factors are explicitly given by

$$\begin{aligned}
F_{1,\gamma}^{(a)}(x, y) &= \left[ \frac{1}{18} \frac{\partial^3 \varrho_{3,1}}{\partial y^3} - \frac{1}{4} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} - \frac{5}{6} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y), \\
F_{2,\gamma}^{(a)}(x, y) &= \left[ -\frac{2}{3} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{10}{3} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y), \\
F_{1,g}^{(a)}(x, y) &= \left[ \frac{1}{12} \frac{\partial^3 \varrho_{3,1}}{\partial y^3} - \frac{1}{2} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y), \\
F_{2,g}^{(a)}(x, y) &= \left[ -\frac{\partial^2 \varrho_{2,1}}{\partial y^2} + 2 \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y).
\end{aligned} \tag{58}$$

Here, the function  $\varrho_{m,n}(x, y)$  is defined through

$$\varrho_{m,n}(x, y) = \frac{x^m \ln^n x - y^m \ln^n y}{x - y}. \tag{59}$$

When Fig.(a1)and (a2), are composed by the charged 5/3 quarks, the corrections to the coefficients at the EW scale  $\mu_{\text{EW}}$  are formulated as

$$\begin{aligned}
C_{7\gamma}^{(a)\frac{5}{3}} &= \frac{ie^3}{16\pi^2\mu_{\text{EW}}^2 s_W^2} \sum_{\beta=1}^{\infty} \left\{ \left( \xi_{W^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \xi_{W^\pm}^{L(5/3)} \right)_{\beta,t} F_{1,\gamma}^{(a)\frac{5}{3}}(x_{H_\beta}, x_{W^\pm}) \right. \\
&\quad + \frac{m_{H_\beta}}{m_t} \left( \xi_{W^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \xi_{W^\pm}^{R(5/3)} \right)_{\beta,t} F_{2,\gamma}^{(a)\frac{5}{3}}(x_{H_\beta}, x_{W^\pm}) \\
&\quad + \sum_{\alpha=1}^{\infty} \left( \xi_{W_{H_\alpha}^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \xi_{W_{H_\alpha}^\pm}^{L(5/3)} \right)_{\beta,t} F_{1,\gamma}^{(a)\frac{5}{3}}(x_{H_\beta}, x_{W_{H_\alpha}^\pm}) \\
&\quad \left. + \frac{m_{H_\beta}}{m_t} \sum_{\alpha=1}^{\infty} \left( \xi_{W_{H_\alpha}^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \xi_{W_{H_\alpha}^\pm}^{R(5/3)} \right)_{\beta,t} F_{2,\gamma}^{(a)\frac{5}{3}}(x_{H_\beta}, x_{W_{H_\alpha}^\pm}) \right\}, \\
C_{8G}^{(a)\frac{5}{3}} &= \frac{ie^2 g_s T^a}{16\pi^2\mu_{\text{EW}}^2 s_W^2} \sum_{\beta=1}^{\infty} \left\{ \left( \xi_{W^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \xi_{W^\pm}^{L(5/3)} \right)_{\beta,t} F_{1,g}^{(a)\frac{5}{3}}(x_{H_\beta}, x_{W^\pm}) \right. \\
&\quad + \frac{m_{H_\beta}}{m_t} \left( \xi_{W^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \xi_{W^\pm}^{R(5/3)} \right)_{\beta,t} F_{2,g}^{(a)\frac{5}{3}}(x_{H_\beta}, x_{W^\pm}) \\
&\quad + \sum_{\alpha=1}^{\infty} \left( \xi_{W_{H_\alpha}^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \xi_{W_{H_\alpha}^\pm}^{L(5/3)} \right)_{\beta,t} F_{1,g}^{(a)\frac{5}{3}}(x_{H_\beta}, x_{W_{H_\alpha}^\pm}) \\
&\quad \left. + \frac{m_{H_\beta}}{m_t} \sum_{\alpha=1}^{\infty} \left( \xi_{W_{H_\alpha}^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \xi_{W_{H_\alpha}^\pm}^{R(5/3)} \right)_{\beta,t} F_{2,g}^{(a)\frac{5}{3}}(x_{H_\beta}, x_{W_{H_\alpha}^\pm}) \right\}, \\
\tilde{C}_{7\gamma}^{(a)\frac{5}{3}} &= C_{7\gamma}^{(a)\frac{5}{3}} \left( \xi_{W^\pm}^{L(5/3)} \leftrightarrow \xi_{W^\pm}^{R(5/3)}, \xi_{W_{H_\alpha}^\pm}^{L(5/3)} \leftrightarrow \xi_{W_{H_\alpha}^\pm}^{R(5/3)} \right), \\
\tilde{C}_{8G}^{(a)\frac{5}{3}} &= C_{8G}^{(a)\frac{5}{3}} \left( \xi_{W^\pm}^{L(5/3)} \leftrightarrow \xi_{W^\pm}^{R(5/3)}, \xi_{W_{H_\alpha}^\pm}^{L(5/3)} \leftrightarrow \xi_{W_{H_\alpha}^\pm}^{R(5/3)} \right) \quad (60)
\end{aligned}$$

And the form factors are

$$\begin{aligned}
F_{1,\gamma}^{(a)\frac{5}{3}}(x, y) &= \left[ \frac{5}{36} \frac{\partial^3 \varrho_{3,1}}{\partial y^3} - \frac{1}{4} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} - \frac{4}{3} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y), \\
F_{2,\gamma}^{(a)\frac{5}{3}}(x, y) &= \left[ -\frac{5}{3} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{16}{3} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y), \\
F_{1,g}^{(a)\frac{5}{3}}(x, y) &= \left[ \frac{1}{12} \frac{\partial^3 \varrho_{3,1}}{\partial y^3} - \frac{1}{2} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y), \\
F_{2,g}^{(a)\frac{5}{3}}(x, y) &= \left[ -\frac{\partial^2 \varrho_{2,1}}{\partial y^2} + 2 \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y). \quad (61)
\end{aligned}$$

Using  $\xi^{L,R}$  in appendix, we could see that in Fig.(a1)(a2), the relevant coefficients  $C_{7\gamma}^{(a)}$  and  $C_{8g}^{(a)}$  are at order  $O(\frac{v^4}{\Lambda_{KK}^4})$ , so we ignore them.

2. *Fig.(b1) and (b2) with KK mode propagators*

Similarly, we can write down the corrections to Wilson coefficients at the EW scale  $\mu_{\text{EW}}$  from Fig.(b1),(b2), which are composed by the KK mode of charged Goldstone and the KK mode of charged  $-1/3$  quarks

$$\begin{aligned}
C_{7\gamma}^{(b)} &= \frac{ie}{8\pi^2\mu_{\text{EW}}^2} \sum_{\beta=1}^{\infty} \left\{ \left( \eta_{G^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \eta_{G^\pm}^{L(-1/3)} \right)_{\beta,t} F_{1,\gamma}^{(b)}(x_{D_\beta}, x_{W^\pm}) \right. \\
&\quad \left. + \frac{m_{D_\beta}}{m_t} \left( \eta_{G^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \eta_{G^\pm}^{R(-1/3)} \right)_{\beta,t} F_{2,\gamma}^{(b)}(x_{D_\beta}, x_{W^\pm}) \right\}, \\
C_{8G}^{(b)} &= \frac{ig_s T^a}{8\pi^2\mu_{\text{EW}}^2} \sum_{\beta=1}^{\infty} \left\{ \left( \eta_{G^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \eta_{G^\pm}^{L(-1/3)} \right)_{\beta,t} F_{1,g}^{(b)}(x_{D_\beta}, x_{W^\pm}) \right. \\
&\quad \left. + \frac{m_{D_\beta}}{m_t} \left( \eta_{G^\pm}^{L(-1/3)} \right)_{c,\beta}^\dagger \left( \eta_{G^\pm}^{R(-1/3)} \right)_{\beta,t} F_{2,g}^{(b)}(x_{D_\beta}, x_{W^\pm}) \right\}, \\
\tilde{C}_{7\gamma}^{(b)} &= C_{7\gamma}^{(b)} \left( \eta_{G^\pm}^{L(-1/3)} \leftrightarrow \eta_{G^\pm}^{R(-1/3)} \right), \\
\tilde{C}_{8G}^{(b)} &= C_{8G}^{(b)} \left( \eta_{G^\pm}^{L(-1/3)} \leftrightarrow \eta_{G^\pm}^{R(-1/3)} \right), \tag{62}
\end{aligned}$$

and those form factors are given by

$$\begin{aligned}
F_{1,\gamma}^{(b)}(x, y) &= \left[ \frac{1}{36} \frac{\partial^3 \varrho_{3,1}}{\partial y^3} - \frac{7}{24} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{5}{12} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y), \\
F_{2,\gamma}^{(b)}(x, y) &= \left[ -\frac{1}{6} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{1}{3} \frac{\partial \varrho_{1,1}}{\partial y} - \frac{5}{6} \frac{\partial \varrho_{1,1}}{\partial x} \right] (x, y), \\
F_{1,g}^{(b)}(x, y) &= \left[ \frac{1}{24} \frac{\partial^3 \varrho_{3,1}}{\partial y^3} - \frac{1}{4} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{1}{4} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y), \\
F_{2,g}^{(b)}(x, y) &= \left[ -\frac{1}{4} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{1}{2} \frac{\partial \varrho_{1,1}}{\partial y} - \frac{1}{2} \frac{\partial \varrho_{1,1}}{\partial x} \right] (x, y). \tag{63}
\end{aligned}$$

And the coefficients in Fig.(b1),(b2) composed by the KK mode of charged  $5/3$  quarks are

$$\begin{aligned}
C_{7\gamma}^{(b)\frac{5}{3}} &= \frac{ie}{8\pi^2\mu_{\text{EW}}^2} \sum_{\beta=1}^{\infty} \left\{ \left( \eta_{G^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \eta_{G^\pm}^{L(5/3)} \right)_{\beta,t} F_{1,\gamma}^{(b)\frac{5}{3}}(x_{H_\beta}, x_{W^\pm}) \right. \\
&\quad \left. + \frac{m_{H_\beta}}{m_t} \left( \eta_{G^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \eta_{G^\pm}^{R(5/3)} \right)_{\beta,t} F_{2,\gamma}^{(b)\frac{5}{3}}(x_{H_\beta}, x_{W^\pm}) \right\}, \\
C_{8G}^{(b)\frac{5}{3}} &= \frac{ig_s T^a}{8\pi^2\mu_{\text{EW}}^2} \sum_{\beta=1}^{\infty} \left\{ \left( \eta_{G^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \eta_{G^\pm}^{L(5/3)} \right)_{\beta,t} F_{1,g}^{(b)\frac{5}{3}}(x_{H_\beta}, x_{W^\pm}) \right. \\
&\quad \left. + \frac{m_{H_\beta}}{m_t} \left( \eta_{G^\pm}^{L(5/3)} \right)_{c,\beta}^\dagger \left( \eta_{G^\pm}^{R(5/3)} \right)_{\beta,t} F_{2,g}^{(b)\frac{5}{3}}(x_{H_\beta}, x_{W^\pm}) \right\}, \\
\tilde{C}_{7\gamma}^{(b)\frac{5}{3}} &= C_{7\gamma}^{(b)\frac{5}{3}} \left( \eta_{G^\pm}^{L(5/3)} \leftrightarrow \eta_{G^\pm}^{R(5/3)} \right), \\
\tilde{C}_{8G}^{(b)\frac{5}{3}} &= C_{8G}^{(b)\frac{5}{3}} \left( \eta_{G^\pm}^{L(5/3)} \leftrightarrow \eta_{G^\pm}^{R(5/3)} \right), \tag{64}
\end{aligned}$$

and those form factors are given by

$$\begin{aligned}
F_{1,\gamma}^{(b)\frac{5}{3}}(x, y) &= \left[ \frac{5}{72} \frac{\partial^3 \varrho_{3,1}}{\partial y^3} - \frac{13}{24} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{2}{3} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y), \\
F_{2,\gamma}^{(b)\frac{5}{3}}(x, y) &= \left[ -\frac{5}{12} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{5}{6} \frac{\partial \varrho_{1,1}}{\partial y} - \frac{4}{3} \frac{\partial \varrho_{1,1}}{\partial x} \right] (x, y), \\
F_{1,g}^{(b)\frac{5}{3}}(x, y) &= \left[ \frac{1}{24} \frac{\partial^3 \varrho_{3,1}}{\partial y^3} - \frac{1}{4} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{1}{4} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y), \\
F_{2,g}^{(b)\frac{5}{3}}(x, y) &= \left[ -\frac{1}{4} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{1}{2} \frac{\partial \varrho_{1,1}}{\partial y} - \frac{1}{2} \frac{\partial \varrho_{1,1}}{\partial x} \right] (x, y). \tag{65}
\end{aligned}$$

The corrections to relevant Wilson coefficients with the KK exciting modes of virtual charged  $-1/3$  quarks, are analogously formulated to the order  $\mathcal{O}(v^2/\Lambda_{KK}^2)$  as

$$\begin{aligned}
C_{7\gamma}^{(b)} &= \frac{11ie}{576\pi^2\Lambda_{KK}^2} \sum_{i,j,k=1}^3 (U_L^{(0)})_{ci}^\dagger [f_{(++)}^{L,c_B^i}(0,1)] Y_{ik}^d [\Sigma_{(\mp\mp)}^{R,c_T^k}(1,1)] Y_{kj}^{d\dagger} [f_{(++)}^{L,c_B^j}(0,1)] (U_L^{(0)})_{jt} + \mathcal{O}(\frac{v^4}{\Lambda_{KK}^4}), \\
C_{8G}^{(b)} &= \frac{ig_s T^a}{192\pi^2\Lambda_{KK}^2} \sum_{i,j,k=1}^3 (U_L^{(0)})_{ci}^\dagger [f_{(++)}^{L,c_B^i}(0,1)] Y_{ik}^d [\Sigma_{(\mp\mp)}^{R,c_T^k}(1,1)] Y_{kj}^{d\dagger} [f_{(++)}^{L,c_B^j}(0,1)] (U_L^{(0)})_{jt} + \mathcal{O}(\frac{v^4}{\Lambda_{KK}^4}), \\
\tilde{C}_{7\gamma}^{(b)} &= \frac{11ie}{576\pi^2\Lambda_{KK}^2} \sum_{i,j,k=1}^3 (U_R^{(0)})_{ci}^\dagger [f_{(++)}^{R,c_S^i}(0,1)] Y_{ik}^{u\dagger} [\Sigma_{(\pm\pm)}^{L,c_B^k}(1,1)] Y_{kj}^u [f_{(++)}^{R,c_S^j}(0,1)] (U_R^{(0)})_{jt} + \mathcal{O}(\frac{v^4}{\Lambda_{KK}^4}), \\
\tilde{C}_{8G}^{(b)} &= \frac{ig_s T^a}{192\pi^2\Lambda_{KK}^2} \sum_{i,j,k=1}^3 (U_R^{(0)})_{ci}^\dagger [f_{(++)}^{R,c_S^i}(0,1)] Y_{ik}^{u\dagger} [\Sigma_{(\pm\pm)}^{L,c_B^k}(1,1)] Y_{kj}^u [f_{(++)}^{R,c_S^j}(0,1)] (U_R^{(0)})_{jt} + \mathcal{O}(\frac{v^4}{\Lambda_{KK}^4}). \tag{66}
\end{aligned}$$

and for charge 5/3 quarks

$$\begin{aligned}
C_{7\gamma}^{(b)\frac{5}{3}} &= \frac{7ie}{288\pi^2\Lambda_{KK}^2} \sum_{i,j,k=1}^3 (U_L^{(0)})_{ci}^\dagger [f_{(++)}^{L,c_B^i}(0,1)] Y_{ik}^d [2\Sigma_{(\pm\mp)}^{R,c_T^k}(1,1)] Y_{kj}^{d\dagger} [f_{(++)}^{L,c_B^j}(0,1)] (U_L^{(0)})_{jt} + \mathcal{O}(\frac{v^4}{\Lambda_{KK}^4}) , \\
C_{8G}^{(b)\frac{5}{3}} &= \frac{ig_s T^a}{192\pi^2\Lambda_{KK}^2} \sum_{i,j,k=1}^3 (U_L^{(0)})_{ci}^\dagger [f_{(++)}^{L,c_B^i}(0,1)] Y_{ik}^d [2\Sigma_{(\pm\mp)}^{R,c_T^k}(1,1)] Y_{kj}^{d\dagger} [f_{(++)}^{L,c_B^j}(0,1)] (U_L^{(0)})_{jt} + \mathcal{O}(\frac{v^4}{\Lambda_{KK}^4}) , \\
\tilde{C}_{7\gamma}^{(b)\frac{5}{3}} &= \frac{7ie}{288\pi^2\Lambda_{KK}^2} \sum_{i,j,k=1}^3 (U_R^{(0)})_{ci}^\dagger [f_{(++)}^{R,c_S^i}(0,1)] Y_{ik}^{u\dagger} [\Sigma_{(\mp\pm)}^{L,c_B^k}(1,1)] Y_{kj}^u [f_{(++)}^{R,c_S^j}(0,1)] (U_R^{(0)})_{jt} + \mathcal{O}(\frac{v^4}{\Lambda_{KK}^4}) , \\
\tilde{C}_{8G}^{(b)\frac{5}{3}} &= \frac{ig_s T^a}{192\pi^2\Lambda_{KK}^2} \sum_{i,j,k=1}^3 (U_R^{(0)})_{ci}^\dagger [f_{(++)}^{R,c_S^i}(0,1)] Y_{ik}^{u\dagger} [\Sigma_{(\mp\pm)}^{L,c_B^k}(1,1)] Y_{kj}^u [f_{(++)}^{R,c_S^j}(0,1)] (U_R^{(0)})_{jt} + \mathcal{O}(\frac{v^4}{\Lambda_{KK}^4}) .
\end{aligned} \tag{67}$$

### 3. Fig.(c) with KK mode propagators

For the Feynman diagrams drawn in Fig.(c), intermediate virtual particles involve the KK mode of neutral gauge bosons  $Z$ ,  $Z_{H_\alpha}$ ,  $\gamma_{(n)}$ , and the KK mode of charged 2/3 quarks, the corresponding corrections to Wilson coefficients at electroweak scale are expressed as

$$\begin{aligned}
C_{7\gamma}^{(c)} &= \frac{ie^3}{48\pi^2\mu_{EW}^2 s_W^2 c_W^2} \sum_{\beta=1}^{\infty} \left\{ \left( \xi_Z^{L(2/3)} \right)_{c,\beta}^\dagger \left( \xi_Z^{L(2/3)} \right)_{\beta,t} F_{1,g}^{(a)}(x_{U_\beta}, x_Z) \right. \\
&\quad + \frac{m_{U_\beta}}{m_t} \left( \xi_Z^{L(2/3)} \right)_{c,\beta}^\dagger \left( \xi_Z^{R(2/3)} \right)_{\beta,t} F_{2,g}^{(a)}(x_{U_\beta}, x_Z) \\
&\quad + \sum_{\alpha=1}^{\infty} \left( \xi_{Z_{H_\alpha}}^{L(2/3)} \right)_{c,\beta}^\dagger \left( \xi_{Z_{H_\alpha}}^{L(2/3)} \right)_{\beta,t} F_{1,g}^{(a)}(x_{U_\beta}, x_{Z_{H_\alpha}}) \\
&\quad + \frac{m_{U_\beta}}{m_t} \sum_{\alpha=1}^{\infty} \left( \xi_{Z_{H_\alpha}}^{L(2/3)} \right)_{c,\beta}^\dagger \left( \xi_{Z_{H_\alpha}}^{R(2/3)} \right)_{\beta,t} F_{2,g}^{(a)}(x_{U_\beta}, x_{Z_{H_\alpha}}) \Big\} \\
&\quad + \frac{2e^2}{27\mu_{EW}^2} \sum_{n=1}^{\infty} \sum_{\beta=1}^{\infty} \left\{ \left( \xi_{\gamma_{(n)}}^{L(2/3)} \right)_{c,\beta}^\dagger \left( \xi_{\gamma_{(n)}}^{L(2/3)} \right)_{\beta,t} F_{1,g}^{(a)}(x_{U_\beta}, x_{\gamma_{(n)}}) \right. \\
&\quad + \frac{m_{U_\beta}}{m_t} \left( \xi_{\gamma_{(n)}}^{L(2/3)} \right)_{c,\beta}^\dagger \left( \xi_{\gamma_{(n)}}^{R(2/3)} \right)_{\beta,t} F_{2,g}^{(a)}(x_{U_\beta}, x_{\gamma_{(n)}}) \Big\} , \\
C_{8G}^{(c)} &= \frac{3g_s T^a}{2e} C_{7\gamma}^{(c)} , \\
\tilde{C}_{7\gamma}^{(c)} &= C_{7\gamma}^{(c)} \left( \xi_Z^{L(2/3)} \leftrightarrow \xi_Z^{R(2/3)}, \xi_{Z_{H_\alpha}}^{L(2/3)} \leftrightarrow \xi_{Z_{H_\alpha}}^{R(2/3)}, \xi_{\gamma_{(n)}}^{L(2/3)} \leftrightarrow \xi_{\gamma_{(n)}}^{R(2/3)} \right) , \\
\tilde{C}_{8G}^{(c)} &= C_{8G}^{(c)} \left( \xi_Z^{L(2/3)} \leftrightarrow \xi_Z^{R(2/3)}, \xi_{Z_{H_\alpha}}^{L(2/3)} \leftrightarrow \xi_{Z_{H_\alpha}}^{R(2/3)}, \xi_{\gamma_{(n)}}^{L(2/3)} \leftrightarrow \xi_{\gamma_{(n)}}^{R(2/3)} \right) .
\end{aligned} \tag{68}$$



The corrections to relevant Wilson coefficients from Fig.(c) are analogously formulated to the order  $\mathcal{O}(v^2/\Lambda_{KK}^2)$  as

$$\begin{aligned}
C_{7\gamma}^{(c)} &= \frac{32ie^3}{9\pi\Lambda_{KK}^2(kr\epsilon)^2} \sum_{i,j,k=1}^3 \left\{ \frac{m_{u_k}}{m_t} (U_L^{(0)})_{ci}^\dagger (U_L^{(0)})_{ik} (U_R^{(0)})_{kj}^\dagger (U_R^{(0)})_{jt} \right. \\
&\quad \times \int_\epsilon^1 dt \int_\epsilon^1 dt' \left( \frac{1}{c_w^2} [\Sigma_{(++)}^G(t, t')] + \frac{3-2s_w^2}{c_w^2(1-2s_w^2)} [\Sigma_{(-+)}^G(t, t')] \right) \\
&\quad \times [f_{(++)}^{L,c_B^i}(0, t)]^2 [f_{(++)}^{R,c_S^j}(0, t')]^2 \Big\} + \mathcal{O}\left(\frac{v^4}{\Lambda_{KK}^4}\right), \\
C_{8G}^{(c)} &= \frac{3g_s T^a}{2e} C_{7\gamma}^{(c)}, \\
\tilde{C}_{7\gamma}^{(c)} &= \frac{32ie^3}{9\pi\Lambda_{KK}^2(kr\epsilon)^2} \sum_{i,j,k=1}^3 \left\{ \frac{m_{u_k}}{m_t} (U_R^{(0)})_{ci}^\dagger (U_R^{(0)})_{ik} (U_L^{(0)})_{kj}^\dagger (U_L^{(0)})_{jt} \right. \\
&\quad \times \int_\epsilon^1 dt \int_\epsilon^1 dt' \left( \frac{1}{c_w^2} [\Sigma_{(++)}^G(t, t')] + \frac{3-2s_w^2}{c_w^2(1-2s_w^2)} [\Sigma_{(-+)}^G(t, t')] \right) \\
&\quad \times [f_{(++)}^{L,c_B^i}(0, t)]^2 [f_{(++)}^{R,c_S^j}(0, t')]^2 \Big\} + \mathcal{O}\left(\frac{v^4}{\Lambda_{KK}^4}\right), \\
\tilde{C}_{8G}^{(c)} &= \frac{3g_s T^a}{2e} \tilde{C}_{7\gamma}^{(c)}. \tag{69}
\end{aligned}$$

#### 4. Fig.(d1) and (d2) with KK mode propagators

Correspondingly, the contributions to Wilson coefficients at electroweak scale from Fig.(d) with the KK mode of gluon, and the KK mode of charged 2/3 quarks are

$$\begin{aligned}
C_{7\gamma}^{(d)} &= \frac{ie g_s^2}{9\pi^2 \mu_{EW}^2} \sum_{n=1}^{\infty} \sum_{\beta=1}^{\infty} \left\{ \left( \xi_{g(n)}^{L(2/3)} \right)_{c,\beta}^\dagger \left( \xi_{g(n)}^{L(2/3)} \right)_{\beta,t} F_{1,g}^{(a)}(x_{U_\beta}, x_{g(n)}) \right. \\
&\quad \left. + \frac{m_{U_\beta}}{m_t} \left( \xi_{g(n)}^{L(2/3)} \right)_{c,\beta}^\dagger \left( \xi_{g(n)}^{R(2/3)} \right)_{\beta,t} F_{2,g}^{(a)}(x_{U_\beta}, x_{g(n)}) \right\}, \\
C_{8G}^{(d)} &= \frac{ig_s^3 T^a}{8\pi^2 \mu_{EW}^2} \sum_{n=1}^{\infty} \sum_{\beta=1}^{\infty} \left\{ \left( \xi_{g(n)}^{L(2/3)} \right)_{c,\beta}^\dagger \left( \xi_{g(n)}^{L(2/3)} \right)_{\beta,t} F_{1,g}^{(d)}(x_{U_\beta}, x_{g(n)}) \right. \\
&\quad \left. + \frac{m_{U_\beta}}{m_t} \left( \xi_{g(n)}^{L(2/3)} \right)_{c,\beta}^\dagger \left( \xi_{g(n)}^{R(2/3)} \right)_{\beta,t} F_{2,g}^{(d)}(x_{U_\beta}, x_{g(n)}) \right\}, \\
\tilde{C}_{7\gamma}^{(d)} &= C_{7\gamma}^{(d)} \left( \xi_{g(n)}^{L(2/3)} \leftrightarrow \xi_{g(n)}^{R(2/3)} \right), \\
\tilde{C}_{8G}^{(d)} &= C_{8G}^{(d)} \left( \xi_{g(n)}^{L(2/3)} \leftrightarrow \xi_{g(n)}^{R(2/3)} \right), \tag{70}
\end{aligned}$$

where the form factors are defined as

$$\begin{aligned}
F_{1,g}^{(d)}(x, y) &= \left[ -\frac{5}{36} \frac{\partial^3 \varrho_{3,1}}{\partial y^3} - \frac{3}{8} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} + \frac{1}{12} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y) , \\
F_{2,g}^{(d)}(x, y) &= \left[ \frac{5}{3} \frac{\partial^2 \varrho_{2,1}}{\partial y^2} - \frac{1}{3} \frac{\partial \varrho_{1,1}}{\partial y} \right] (x, y) .
\end{aligned} \tag{71}$$

Using the coefficients in the appendix , the Wilson coefficients could be approximated by

$$\begin{aligned}
C_{7\gamma}^{(d)} &= \frac{32ie g_s^2}{9\pi \Lambda_{KK}^2 (kr\epsilon)^2} \sum_{i,j,k=1}^3 \left\{ \frac{m_{u_k}}{m_t} (U_L^{(0)})_{ci}^\dagger (U_L^{(0)})_{ik} (U_R^{(0)})_{kj}^\dagger (U_R^{(0)})_{jt} \right. \\
&\quad \times \int_\epsilon^1 dt \int_\epsilon^1 dt' [\Sigma_{(++)}^G(t, t')] [f_{(++)}^{L, c_B^i}(0, t)]^2 [f_{(++)}^{R, c_S^j}(0, t')]^2 \left. \right\} + \mathcal{O}\left(\frac{v^4}{\Lambda_{KK}^4}\right) , \\
C_{8G}^{(d)} &= \frac{16ie g_s^3 T^a}{3\pi \Lambda_{KK}^2 (kr\epsilon)^2} \sum_{i,j,k=1}^3 \left\{ \frac{m_{u_k}}{m_t} (U_L^{(0)})_{ci}^\dagger (U_L^{(0)})_{ik} (U_R^{(0)})_{kj}^\dagger (U_R^{(0)})_{jt} \right. \\
&\quad \times \int_\epsilon^1 dt \int_\epsilon^1 dt' [\Sigma_{(++)}^G(t, t')] [f_{(++)}^{L, c_B^i}(0, t)]^2 [f_{(++)}^{R, c_S^j}(0, t')]^2 \left. \right\} + \mathcal{O}\left(\frac{v^4}{\Lambda_{KK}^4}\right) , \\
\tilde{C}_{7\gamma}^{(d)} &= \frac{32ie g_s^2}{9\pi \Lambda_{KK}^2 (kr\epsilon)^2} \sum_{i,j,k=1}^3 \left\{ \frac{m_{u_k}}{m_t} (U_R^{(0)})_{ci}^\dagger (U_R^{(0)})_{ik} (U_L^{(0)})_{kj}^\dagger (U_L^{(0)})_{jt} \right. \\
&\quad \times \int_\epsilon^1 dt \int_\epsilon^1 dt' [\Sigma_{(++)}^G(t, t')] [f_{(++)}^{R, c_S^j}(0, t')]^2 [f_{(++)}^{L, c_B^i}(0, t)]^2 \left. \right\} + \mathcal{O}\left(\frac{v^4}{\Lambda_{KK}^4}\right) , \\
\tilde{C}_{8G}^{(d)} &= \frac{16ie g_s^3 T^a}{3\pi \Lambda_{KK}^2 (kr\epsilon)^2} \sum_{i,j,k=1}^3 \left\{ \frac{m_{u_k}}{m_t} (U_R^{(0)})_{ci}^\dagger (U_R^{(0)})_{ik} (U_L^{(0)})_{kj}^\dagger (U_L^{(0)})_{jt} \right. \\
&\quad \times \int_\epsilon^1 dt \int_\epsilon^1 dt' [\Sigma_{(++)}^G(t, t')] [f_{(++)}^{R, c_S^j}(0, t')]^2 [f_{(++)}^{L, c_B^i}(0, t)]^2 \left. \right\} + \mathcal{O}\left(\frac{v^4}{\Lambda_{KK}^4}\right) \tag{72}
\end{aligned}$$

### 5. Fig.(e) with KK mode propagators

As intermediate virtual particles in Fig.(e) are the KK mode of neutral Higgs/Goldstone, and the KK mode of charged 2/3 quarks, the corresponding corrections to relevant Wilson

coefficients can be written as

$$\begin{aligned}
C_{7\gamma}^{(e)} &= \frac{ie}{12\pi^2\mu_{EW}^2} \sum_{\beta=1}^{\infty} \left\{ \left( \eta_{H_0}^{(2/3)} \right)_{c,\beta}^{\dagger} \left( \eta_{H_0}^{(2/3)} \right)_{\beta,t} F_{1,g}^{(b)}(x_{U_{\beta}}, x_{H_0}) \right. \\
&\quad + \frac{m_{U_{\beta}}}{m_t} \left( \eta_{H_0}^{(2/3)} \right)_{c,\beta}^{\dagger} \left( \eta_{H_0}^{(2/3)} \right)_{\beta,t} F_{2,g}^{(b)}(x_{U_{\beta}}, x_{H_0}) \\
&\quad + \left( \eta_{G_0}^{(2/3)} \right)_{c,\beta}^{\dagger} \left( \eta_{G_0}^{(2/3)} \right)_{\beta,t} F_{1,g}^{(b)}(x_{U_{\beta}}, x_Z) \\
&\quad \left. - \frac{m_{U_{\beta}}}{m_t} \left( \eta_{G_0}^{(2/3)} \right)_{c,\beta}^{\dagger} \left( \eta_{G_0}^{(2/3)} \right)_{\beta,t} F_{2,g}^{(b)}(x_{U_{\beta}}, x_Z) \right\}, \\
C_{8G}^{(e)} &= \frac{3g_s T^a}{2e} C_{7\gamma}^{(e)}, \\
\tilde{C}_{7\gamma}^{(e)} &= C_{7\gamma}^{(e)} \left( \eta_{H_0}^{(2/3)} \leftrightarrow (\eta_{H_0}^{(2/3)})^{\dagger}, \eta_{G_0}^{(2/3)} \leftrightarrow -(\eta_{G_0}^{(2/3)})^{\dagger} \right), \\
\tilde{C}_{8G}^{(e)} &= C_{8G}^{(e)} \left( \eta_{H_0}^{(2/3)} \leftrightarrow (\eta_{H_0}^{(2/3)})^{\dagger}, \eta_{G_0}^{(2/3)} \leftrightarrow -(\eta_{G_0}^{(2/3)})^{\dagger} \right). \tag{73}
\end{aligned}$$

Using the coefficients in the appendix, we can approximate  $C_{7\gamma}$  and  $C_{8g}$  as

$$\begin{aligned}
C_{7\gamma}^{(e)} &= \frac{ie}{144\pi^2\Lambda_{KK}^2} \sum_{i,j,k=1}^3 \left\{ (U_L^{(0)})_{ci}^{\dagger} f_{(++)}^{L,c_B^i}(0,1) Y_{ik}^u([\Sigma_{(\mp\mp)}^{R,c_S^k}(1,1)]) \right. \\
&\quad \left. \times Y_{kj}^{u\dagger} f_{(++)}^{L,c_B^j}(0,1) (U_L^{(0)})_{jt} \right\} + \mathcal{O}\left(\frac{v^4}{\Lambda_{KK}^4}\right), \\
C_{8G}^{(e)} &= \frac{3g_s T^a}{2e} C_{7\gamma}^{(e)}, \\
\tilde{C}_{7\gamma}^{(e)} &= \frac{ie}{144\pi^2\Lambda_{KK}^2} \sum_{i,j,k=1}^3 \left\{ (U_R^{(0)})_{ci}^{\dagger} f_{(++)}^{R,c_S^i}(0,1) Y_{ik}^{u\dagger}([\Sigma_{(\pm\pm)}^{L,c_B^k}(1,1)] + [\Sigma_{(\mp\pm)}^{L,c_B^k}(1,1)]) \right. \\
&\quad \left. \times Y_{kj}^{u\dagger} f_{(++)}^{R,c_S^j}(0,1) (U_R^{(0)})_{jt} \right\} + \mathcal{O}\left(\frac{v^4}{\Lambda_{KK}^4}\right), \\
\tilde{C}_{8G}^{(e)} &= \frac{3g_s T^a}{2e} \tilde{C}_{7\gamma}^{(e)}. \tag{74}
\end{aligned}$$

#### IV. NUMERICAL ANALYSIS

In general case, the partial widths of these processes are [1]

$$\begin{aligned}
\Gamma(t \rightarrow c\gamma) &= \frac{m_t^3}{16\pi}(m_c^2|F_{TL}^\gamma|^2 + m_t^2|F_{TR}^\gamma|^2) \\
\Gamma(t \rightarrow cg) &= \frac{m_t^3 C_F}{16\pi}(m_c^2|F_{TL}^g|^2 + m_t^2|F_{TR}^g|^2)
\end{aligned} \tag{75}$$

with  $C_F = 4/3$  is a colour factor. Applying Eq(34),(53),(54), we could get that the partial widths both composed by the SM particles and their KK exciting modes has the form,

$$\begin{aligned}
\Gamma(t \rightarrow c\gamma) &= \frac{m_t^3}{16\pi}(m_c^2|F_{TL}^\gamma + \tilde{C}_{7\gamma}|^2 + m_t^2|F_{TR}^\gamma + C_{7\gamma}|^2) \\
\Gamma(t \rightarrow cg) &= \frac{m_t^3 C_F}{16\pi}(m_c^2|F_{TL}^g + \tilde{C}_{8g}|^2 + m_t^2|F_{TR}^g + C_{8g}|^2)
\end{aligned} \tag{76}$$

We compute the branching ratio by taking the SM charged-current two-body decay  $t \rightarrow bW$  to be the dominant  $t$ -quark decay mode, which is  $\Gamma(t \rightarrow bW^+) = 1.466|V_{tb}|^2$  for  $m_t = 172\text{GeV}$ ,  $m_W = 80.399\text{GeV}$ . We will then approximate the branching ratio by

$$\begin{aligned}
Br(t \rightarrow c\gamma) &= \frac{\Gamma(t \rightarrow c\gamma)}{\Gamma(t \rightarrow bW^+)} \\
Br(t \rightarrow cg) &= \frac{\Gamma(t \rightarrow cg)}{\Gamma(t \rightarrow bW^+)}
\end{aligned} \tag{77}$$

The input parameters which we are going to use in the numerical computations are :  $m_W = 80.399\text{GeV}$ ,  $m_Z = 90.19\text{GeV}$ ,  $m_c = 1.27\text{GeV}$ ,  $m_t = 172\text{GeV}$ . We choose the Yukawa couplings  $Y_{ij}^u = 0.01$  ( $i \neq j$ ,  $i, j = 1, 2, 3$ ),  $Y_{21}^d = Y_{31}^d = Y_{32}^d = 0.01$ . And the Wolfenstein parameters of the CKM matrix are  $\lambda = 0.22$ ,  $A = 0.81$ ,  $\bar{\rho} = 0.13$ ,  $\bar{\eta} = 0.34$ .

Assuming anarchic Yukawa couplings, i.e., complex-valued matrices  $Y^u$ ,  $Y^d$  with random elements, we can reproduce the up- and down-type quark mass hierarchies with the ansatz for hierarchical structures of the profiles of zero modes on IR brane[26]:

$$\begin{aligned}
\left[f_{(++)}^{L,c_B^1}(0,1)\right] &< \left[f_{(++)}^{L,c_B^2}(0,1)\right] < \left[f_{(++)}^{L,c_B^3}(0,1)\right], \\
\left[f_{(++)}^{R,c_T^1}(0,1)\right] &< \left[f_{(++)}^{R,c_T^2}(0,1)\right] < \left[f_{(++)}^{R,c_T^3}(0,1)\right], \\
\left[f_{(++)}^{R,c_S^1}(0,1)\right] &< \left[f_{(++)}^{R,c_S^2}(0,1)\right] < \left[f_{(++)}^{R,c_S^3}(0,1)\right].
\end{aligned} \tag{78}$$

Since when  $c > c'$ , we have  $f_{(++)}^{L,c'}(0,1) < f_{(++)}^{L,c}(0,1)$ , and when  $c > c'$ , we have  $f_{(++)}^{R,c'}(0,1) < f_{(++)}^{R,c}(0,1)$ , so the bulk masses  $c_B^i, c_T^i, c_S^i$  must satisfy the relation:

$$\begin{aligned}
c_B^1 &> c_B^2 > c_B^3, \\
c_T^1 &< c_T^2 < c_T^3, \\
c_S^1 &< c_S^2 < c_S^3.
\end{aligned} \tag{79}$$

In order to reduce the number of free parameters in our analysis, we assume

$$\begin{aligned}
c_T^1 = c_S^1 &= -0.75, \quad c_T^2 = c_S^2 = -0.55, \quad c_T^3 = c_S^3 = -0.35; \\
c_B^2 &= -0.5 + c_B^1, \quad c_B^3 = -1 + c_B^1.
\end{aligned} \tag{80}$$

Actually, the assumption on the bulk masses guarantees the profiles of zero modes on IR brane satisfying the hierarchical structures (78). In Fig. 2, we plot  $\text{Br}(t \rightarrow c\gamma)$  varying with the bulk mass  $c_B^1$  under above assumption on the parameter space with the mass scale of low-lying KK states  $\Lambda_{KK} = 1\text{TeV}$ (solid line),  $\Lambda_{KK} = 2\text{TeV}$ (dash line),  $\Lambda_{KK} = 3\text{TeV}$ (dash-dot line). As  $c_B^1 < 0.5$ , the contributions from new physics to  $\text{Br}(t \rightarrow c\gamma)$  decrease steeply, for low-lying KK states  $\Lambda_{KK} = 1, 2, 3\text{TeV}$  respectively. Since the left-handed SM top and charm quarks are contained in bidoublet, so the profiles of  $t$  and  $c$  quarks on IR brane are determined by the bulk mass of  $c_B^i$ . As  $c_B^1 > 1.0$ , the new physics corrections to  $\text{Br}(t \rightarrow c\gamma)$  depend on  $c_B^1$  mildly, and line for  $\Lambda_{KK} = 1, 2, 3\text{TeV}$  are coincide with each other almost.

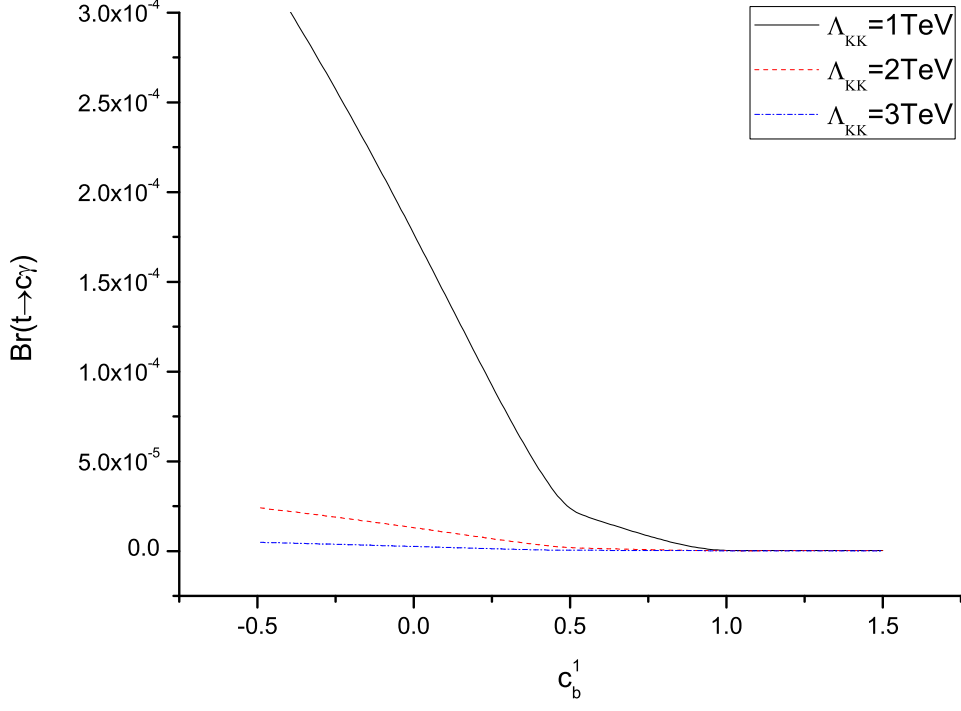


FIG. 2: The branching ratio of  $t \rightarrow c\gamma$  varying with the bulk mass  $c_b^1$ . Here we assuming that  $c_T^1 = c_S^1 = -0.75$ ,  $c_T^2 = c_S^2 = -0.55$ ,  $c_T^3 = c_S^3 = -0.35$ , and  $c_B^2 = -0.5 + c_B^1$ ,  $c_B^3 = -1 + c_B^1$ . The solid line corresponds to the numerical result with  $\Lambda_{KK} = 1\text{TeV}$ , dash line corresponds to the numerical result with  $\Lambda_{KK} = 2\text{TeV}$ , and dash-dot line corresponds to the numerical result with  $\Lambda_{KK} = 3\text{TeV}$ , respectively

To investigate the dependence of  $\text{Br}(t \rightarrow c\gamma)$  on bulk mass of triplet quark  $c_T^1$ , we choose

$$\begin{aligned}
c_S^1 &= -0.75, \quad c_S^2 = -0.55, \quad c_S^3 = -0.35; \\
c_B^1 &= 0.55, \quad c_B^2 = 0.25, \quad c_B^3 = -0.05; \\
c_T^2 &= 0.5 + c_T^1, \quad c_T^3 = 1 + c_T^1,
\end{aligned} \tag{81}$$

The bulk masses satisfying Eq.(81) can guarantee the profiles of zero modes on IR brane satisfy the hierarchical structures(78). Under this choice on parameter space, we draw

$\text{Br}(t \rightarrow c\gamma)$  varying with the bulk mass  $c_T^1$  in Fig.3. Because the profiles of  $t$  and  $c$  quarks on IR brane do not depend on  $c_T^1$ , the dependence of  $\text{Br}(t \rightarrow c\gamma)$  on  $c_T^1$  is very mildly, for  $\Lambda_{KK} = 1, 2, 3\text{TeV}$  respectively.

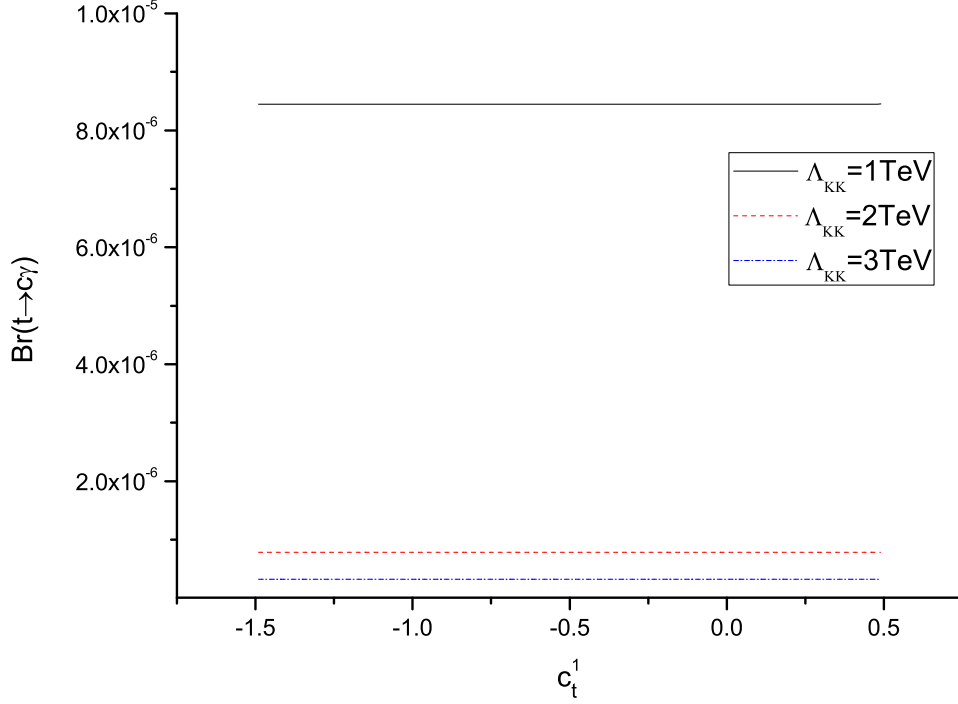


FIG. 3: The branching ratio of  $t \rightarrow c\gamma$  varying with the bulk mass  $c_T^1$ . Here we assuming that  $c_S^1 = -0.75$ ,  $c_S^2 = -0.55$ ,  $c_S^3 = -0.35$ ,  $c_B^1 = 0.55$ ,  $c_B^2 = 0.25$ ,  $c_B^3 = -0.05$ , and  $c_T^2 = 0.5 + c_T^1$ ,  $c_T^3 = 1 + c_T^1$ . The solid line corresponds to the numerical result with  $\Lambda_{KK} = 1\text{TeV}$ , dash line corresponds to the numerical result with  $\Lambda_{KK} = 2\text{TeV}$ , and dash-dot line corresponds to the numerical result with  $\Lambda_{KK} = 3\text{TeV}$ , respectively

Adopting

$$\begin{aligned}
c_T^1 &= -0.75, \quad c_T^2 = -0.55, \quad c_T^3 = -0.35, \\
c_B^1 &= 0.55, \quad c_B^2 = 0.25, \quad c_B^3 = -0.05, \\
c_S^2 &= 0.5 + c_S^1, \quad c_S^3 = 1 + c_S^1,
\end{aligned} \tag{82}$$

to guarantee the profiles of zero modes on IR brane satisfy the hierarchical structures (78). We show the  $\text{Br}(t \rightarrow c\gamma)$  varying with  $c_S^1$  in Fig. 4 for  $\Lambda_{KK} = 1\text{TeV}$ (solid line),  $\Lambda_{KK} = 2\text{TeV}$ (dash line),  $\Lambda_{KK} = 3\text{TeV}$ (dash-dot line), respectively. With the assumption on parameter space, the bulk mass  $c_S^1$  affect the new physics corrections on  $\text{Br}(t \rightarrow c\gamma)$  strongly, since the profiles of right-handed top and charm quarks on IR brane are determined by the bulk mass of singlet  $c_S^i$ .

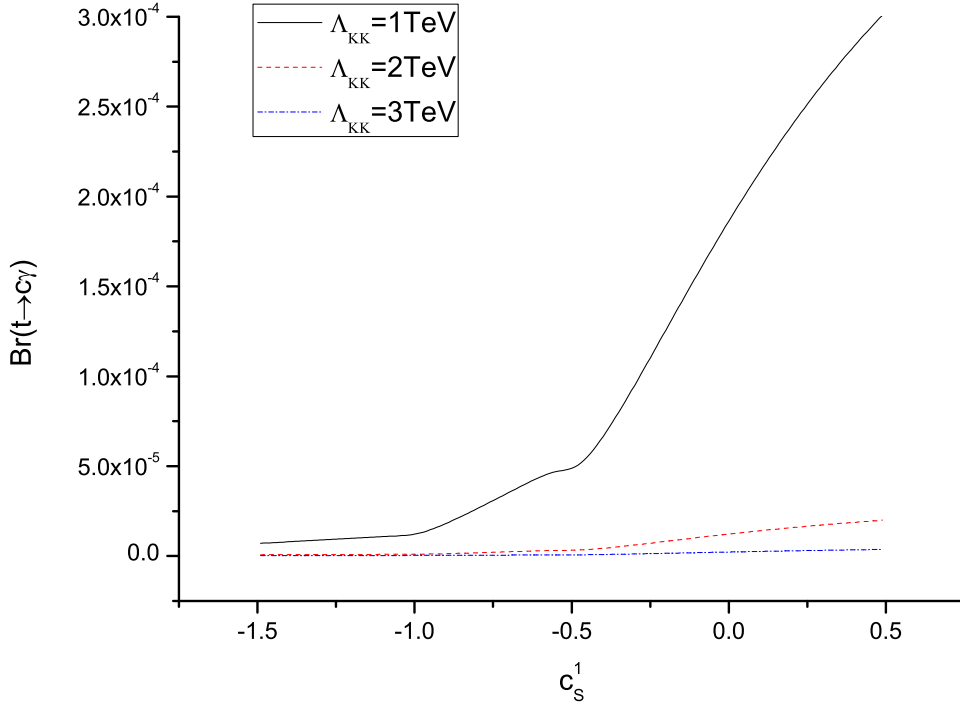


FIG. 4: The branching ratio of  $t \rightarrow c\gamma$  varying with the bulk mass  $c_S^1$ . Here we assuming that  $c_T^1 = -0.75$ ,  $c_T^2 = -0.55$ ,  $c_T^3 = -0.35$ ,  $c_B^1 = 0.55$ ,  $c_B^2 = 0.25$ ,  $c_B^3 = -0.05$ , and  $c_S^2 = 0.5 + c_S^1$ ,  $c_S^3 = 1 + c_S^1$ . The solid line corresponds to the numerical result with  $\Lambda_{KK} = 1\text{TeV}$ , dash line corresponds to the numerical result with  $\Lambda_{KK} = 2\text{TeV}$ , and dash-dot line corresponds to the numerical result with  $\Lambda_{KK} = 3\text{TeV}$ , respectively

In order to appreciate the size of the 3TeV curve, we redraw Fig.2 and Fig. 4 in logarithmic coordinate in Fig.5:



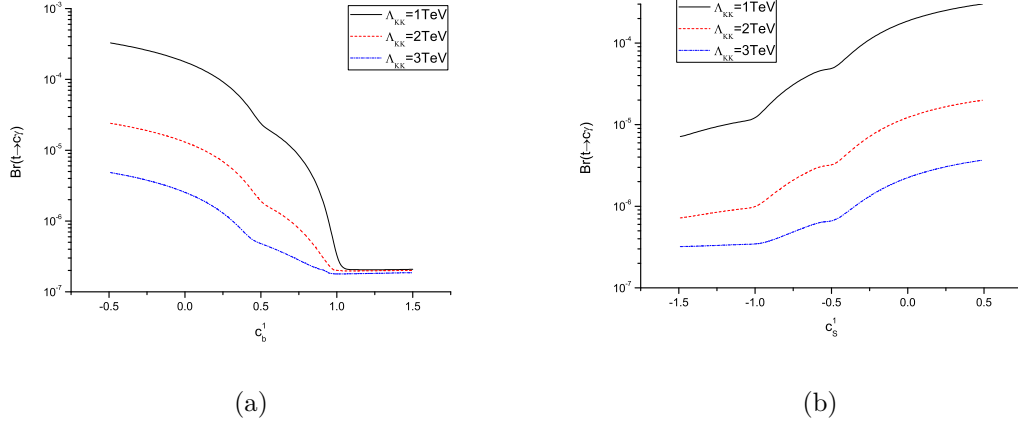


FIG. 5: (a) Redraw Fig.2 in logarithmic coordinate, (b) Redraw Fig.4 in logarithmic coordinate.

In Fig.5(a), when  $c_B^1 \geq 1$ , the three curves become very close, but not coincide with each other, and both in Fig. 5(a) and (b), the branching ratio decreases as  $\Lambda_{KK}$  runs from 1TeV to 3TeV.

Similarly assuming

$$\begin{aligned}
 c_T^1 = c_S^1 = -0.75, \quad c_T^2 = c_S^2 = -0.55, \quad c_T^3 = c_S^3 = -0.35; \\
 c_B^2 = -0.5 + c_B^1, \quad c_B^3 = -1 + c_B^1,
 \end{aligned} \tag{83}$$

to guarantee that the profiles of zero modes on IR brane satisfy the hierarchical structures (78), we plot  $\text{Br}(t \rightarrow cg)$  varying with the bulk mass  $c_B^1$  in Fig.6 under above assumption on the parameter space as the mass scale  $\Lambda_{KK} = 1\text{TeV}$ (solid line),  $\Lambda_{KK} = 2\text{TeV}$ (dash line),  $\Lambda_{KK} = 3\text{TeV}$ (dash-dot line). The contributions from new physics to the branching ratio of  $t \rightarrow cg$  decrease, but slower than which in Fig. 2.

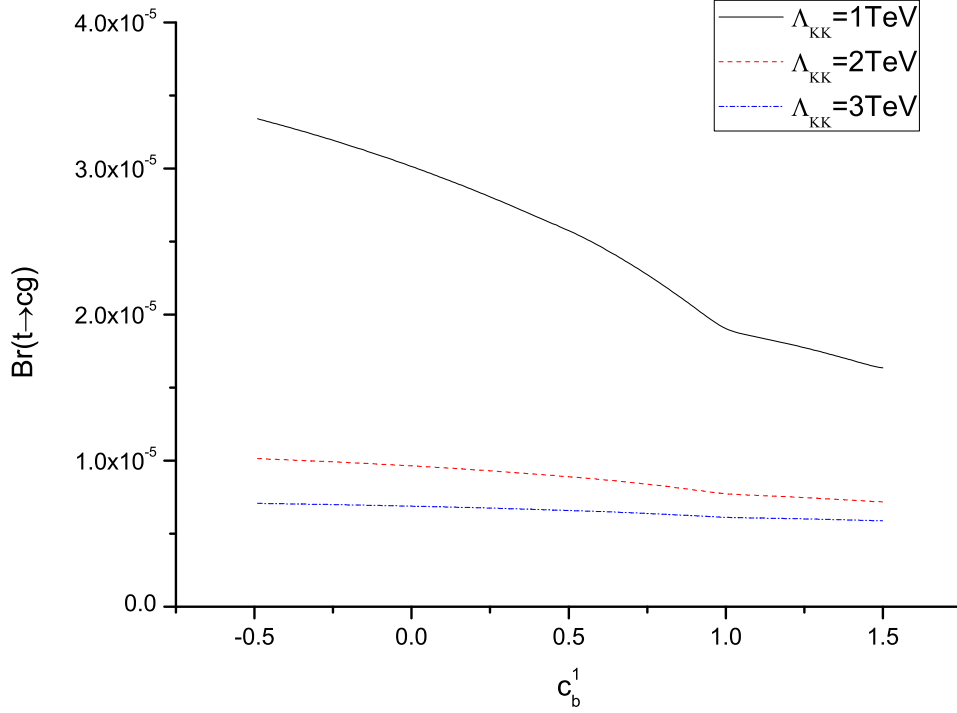


FIG. 6: The branching ratio of  $t \rightarrow cg$  varying with the bulk mass  $c_b^1$ . Here we assuming that  $c_T^1 = c_S^1 = -0.75$ ,  $c_T^2 = c_S^2 = -0.55$ ,  $c_T^3 = c_S^3 = -0.35$ , and  $c_B^2 = -0.5 + c_B^1$ ,  $c_B^3 = -1 + c_B^1$ . The solid line corresponds to the numerical result with  $\Lambda_{KK} = 1\text{TeV}$ , dash line corresponds to the numerical result with  $\Lambda_{KK} = 2\text{TeV}$ , and dash-dot line corresponds to the numerical result with  $\Lambda_{KK} = 3\text{TeV}$ , respectively

Taking

$$\begin{aligned}
c_S^1 &= -0.75, \quad c_S^2 = -0.55, \quad c_S^3 = -0.35; \\
c_B^1 &= 0.55, \quad c_B^2 = 0.25, \quad c_B^3 = -0.05; \\
c_T^2 &= 0.5 + c_T^1, \quad c_T^3 = 1 + c_T^1,
\end{aligned} \tag{84}$$

to guarantee that the profiles of zero modes on IR brane satisfy the hierarchical structures (78). We present  $\text{Br}(t \rightarrow cg)$  varying with the bulk mass  $c_T^1$  in Fig.7, for  $\Lambda_{KK} = 1\text{TeV}$ (solid line),  $\Lambda_{KK} = 2\text{TeV}$ (dash line),  $\Lambda_{KK} = 3\text{TeV}$ (dash-dot line), respectively. Similarly as in

Fig.3, the dependence of the  $t \rightarrow cg$  process on the bulk mass  $c_T^1$  is very mild also, since the profiles of  $t$  and  $c$  quarks on IR brane do not depend on  $c_T^1$ ,

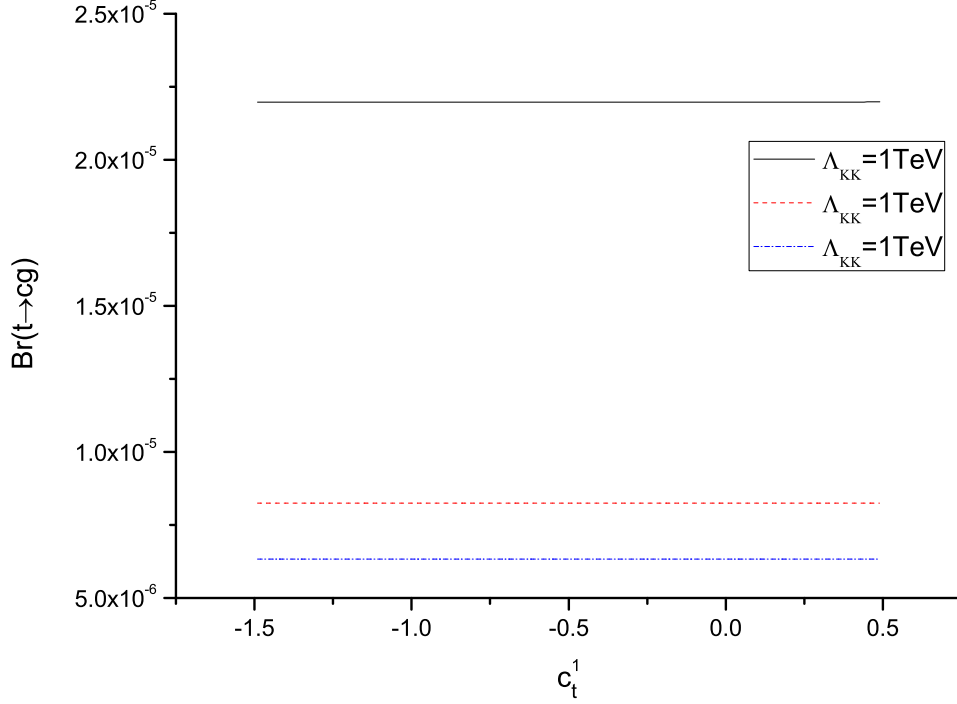


FIG. 7: The branching ratio of  $t \rightarrow cg$  varying with the bulk mass  $c_T^1$ . Here we assuming that  $c_S^1 = -0.75$ ,  $c_S^2 = -0.55$ ,  $c_S^3 = -0.35$ ,  $c_B^1 = 0.55$ ,  $c_B^2 = 0.25$ ,  $c_B^3 = -0.05$ , and  $c_T^2 = 0.5 + c_T^1$ ,  $c_T^3 = 1 + c_T^1$ . The solid line corresponds to the numerical result with  $\Lambda_{KK} = 1\text{TeV}$ , dash line corresponds to the numerical result with  $\Lambda_{KK} = 2\text{TeV}$ , and dash-dot line corresponds to the numerical result with  $\Lambda_{KK} = 3\text{TeV}$ , respectively

Taking

$$\begin{aligned}
c_T^1 &= -0.75, \quad c_T^2 = -0.55, \quad c_T^3 = -0.35, \\
c_B^1 &= 0.55, \quad c_B^2 = 0.25, \quad c_B^3 = -0.05, \\
c_S^2 &= 0.5 + c_S^1, \quad c_S^3 = 1 + c_S^1,
\end{aligned} \tag{85}$$

to guarantee that the profiles of zero modes on IR brane satisfy the hierarchical structures

(78), and we present  $\text{Br}(t \rightarrow cg)$  varying with the bulk mass  $c_S^1$  in Fig.8,. We could see that the contributions from new physics to the  $t \rightarrow cg$  increase quickly when  $c_S^1 > 0.5$ , because of the reason mentioned above, which is similar to that for Fig.4.

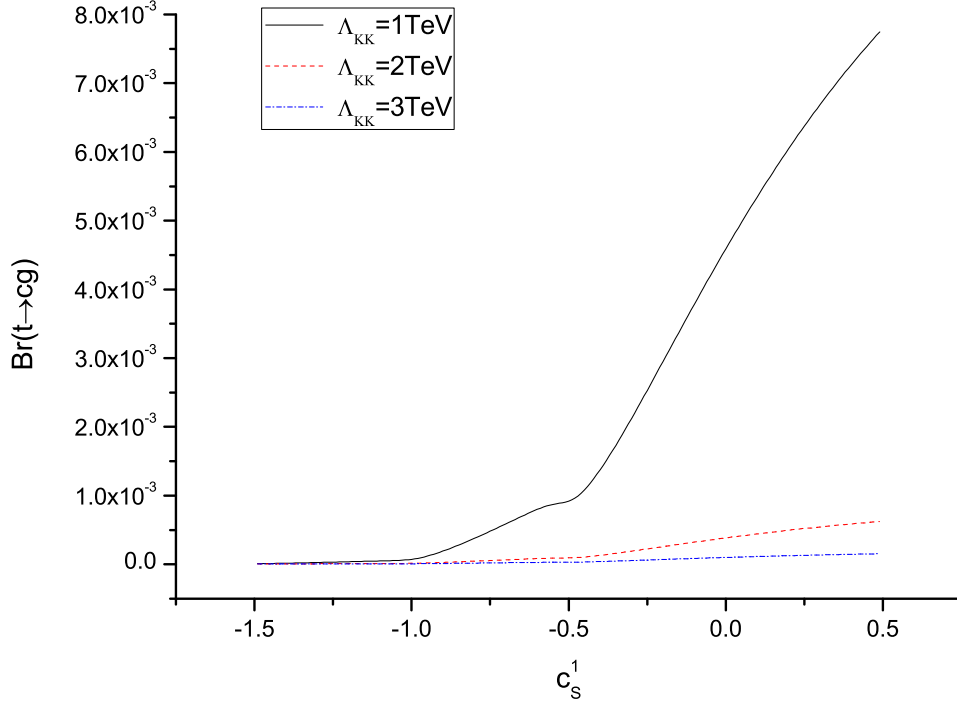


FIG. 8: The branching ratio of  $t \rightarrow cg$  varying with the bulk mass  $c_S^1$ . Here we assuming that  $c_T^1 = -0.75$ ,  $c_T^2 = -0.55$ ,  $c_T^3 = -0.35$ ,  $c_B^1 = 0.55$ ,  $c_B^2 = 0.25$ ,  $c_B^3 = -0.05$ , and  $c_S^2 = 0.5 + c_S^1$ ,  $c_S^3 = 1 + c_S^1$ . The solid line corresponds to the numerical result with  $\Lambda_{KK} = 1\text{TeV}$ , dash line corresponds to the numerical result with  $\Lambda_{KK} = 2\text{TeV}$ , and dash-dot line corresponds to the numerical result with  $\Lambda_{KK} = 3\text{TeV}$ , respectively

In order to appreciate the size of the 3TeV curve, we redraw Fig.6 and Fig. 8 in logarithmic coordinate in Fig. 9:

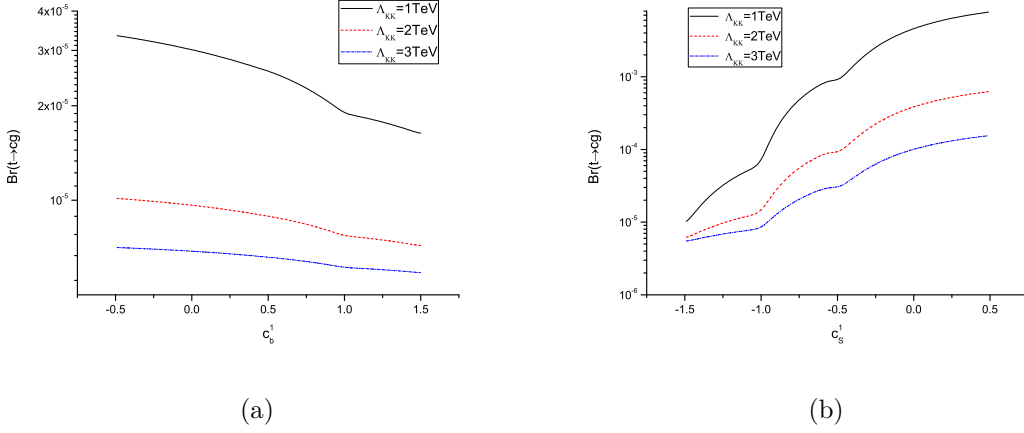


FIG. 9: (a) Redraw Fig.2 in logarithmic coordinate, (b) Redraw Fig.4 in logarithmic coordinate.

Being similar to the case of  $t \rightarrow c\gamma$ , the branching ratio decreases as  $\Lambda_{KK}$  runs from 1 TeV to 3 TeV.

As we could see above, the branching ratio of  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  varying with the bulk mass  $c_B^1$  in Fig. 2 and Fig. 6 are all decrease, since the dominating corrections to the branching ratio of  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  depend on the bulk masses  $c_B^i (i = 1, 2, 3)$  in terms of  $[f_{(++)}^{L, c_B^i}(0, t)][f_{(++)}^{L, c_B^j}(0, t)]$ , with  $f_{(++)}^{L, c}(0, t)$  have the form (Eq. 36 and Eq. 109 of Ref.[37]):

$$f_{(++)}^{L, c}(0, t) = \frac{t^{-c}}{2} \sqrt{\frac{-2(1-2c)\epsilon \ln \epsilon}{\pi(1-\epsilon^{1-2c})}}. \quad (86)$$

In the  $t = 1$  case, the relation between bulk mass  $c$  and  $[f_{(++)}^{L, c}(0, 1)][f_{(++)}^{L, c}(0, 1)]$  are drawn in Fig. 10(a), we could see that the curve is similarly as in Fig. 2 and Fig. 6.

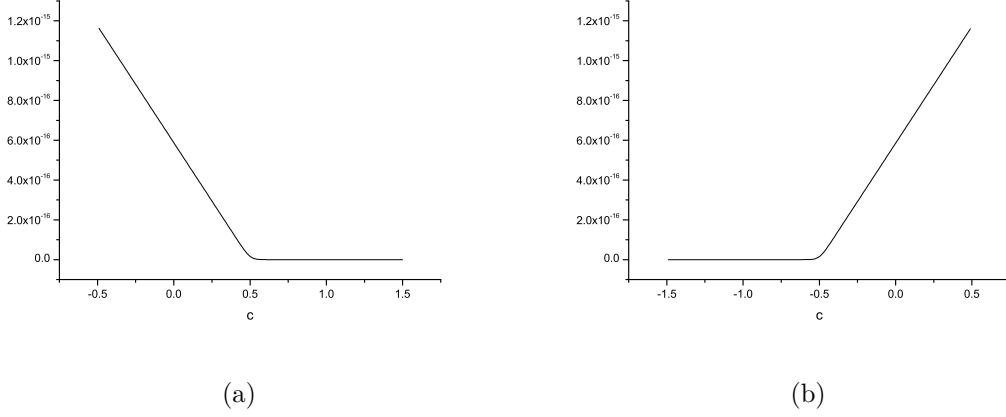


FIG. 10: (a)  $[f_{(++)}^{L,c}(0,1)][f_{(++)}^{L,c}(0,1)]$  varying with the bulk mass  $c$ , (b)  $[f_{(++)}^{R,c}(0,1)][f_{(++)}^{R,c}(0,1)]$  varying with the bulk mass  $c$

Similarly, the curves in Fig. 4 and Fig. 8 increase quickly when  $c_S^1 > 0.5$ , since the dominating corrections to the branching ratio of  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  depend on the bulk masses  $c_S^i (i = 1, 2, 3)$  in terms of  $[f_{(++)}^{R,c_S^i}(0,t)][f_{(++)}^{R,c_S^j}(0,t)]$ , with  $f_{(++)}^{R,c}(0,t)$  have the form:

$$f_{(++)}^{R,c}(0,t) = f_{(++)}^{L,-c}(0,t) = \frac{t^c}{2} \sqrt{\frac{-2(1+2c)\epsilon \ln \epsilon}{\pi(1-\epsilon^{1+2c})}}. \quad (87)$$

In the  $t = 1$  case, the relation between bulk mass  $c$  and  $[f_{(++)}^{R,c}(0,1)][f_{(++)}^{R,c}(0,1)]$  are drawn in Fig. 10(b), the curve increase quickly when  $c_S^1 > 0.5$ , which is similarly as in Fig. 4 and Fig. 8.

## V. CONCLUSION

In the SM, the rare FCNC top decay  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  are suppressed strongly and[8]:

$$\begin{aligned} \text{Br}(t \rightarrow c\gamma) &\approx 4.6 \times 10^{-14}, \\ \text{Br}(t \rightarrow cg) &\approx 4.6 \times 10^{-12}, \end{aligned} \quad (88)$$

which cannot be detected in near future experiment. Considering the constraints from the precise electroweak observations, we investigate the radiative corrections to  $t \rightarrow c\gamma$  and  $t \rightarrow cg$  in warped extra dimensions with the custodial symmetry  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times P_{LR}$ . Since the fifth dimensional profiles depend on the bulk masses  $c_B^i, c_T^i, c_S^i$  ( $i = 1, 2, 3$ ) strongly, we mainly analyze those bulk masses how affecting the corrections to  $\text{Br}(t \rightarrow c\gamma)$  and  $\text{Br}(t \rightarrow cg)$  from new physics. Numerical results indicate that

$$\begin{aligned}\text{Br}(t \rightarrow c\gamma) &\sim 10^{-6}, \\ \text{Br}(t \rightarrow cg) &\sim 10^{-5},\end{aligned}\tag{89}$$

under our assumption on parameter space. With an integrated luminosity of  $100 \text{ fb}^{-1}$ , the estimated precision of LHC to the Br of  $t \rightarrow c\gamma$  is  $1.2 \times 10^{-5}$  [44] and that of the Br to  $t \rightarrow cg$  is about  $2.7 \times 10^{-5}$  [45], respectively. In addition, the TESLA precision to the Br of  $t \rightarrow c\gamma$  can reach  $3.6 \times 10^{-6}$  [46] with a center of mass energy of 800GeV and an integrate luminosity of  $500 \text{ fb}^{-1}$ . We can expect to detect those FCNC processes in near future hopefully.

### Acknowledgments

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## VI. THE COUPLINGS BETWEEN BOSONS AND QUARKS AT THE ORDER

$\mathcal{O}(v^2/\Lambda_{KK}^2)$

The relevant nontrivial couplings in Fig.(a1),(a2) with charged  $-1/3$  quarks could be approached to the order  $\mathcal{O}(v^2/\Lambda_{KK}^2)$  as:

$$\begin{aligned}
(\xi_{W^\pm}^{L(-1/3)})_{b,t} &= (V_{CKM}^{(0)})_{bt}^\dagger + (V_{CKM}^{(0)\dagger} \delta Z_L^u)_{bt} + (V_{CKM}^{(0)} \delta Z_L^d)_{bt}^\dagger - \frac{v^2}{4\Lambda_{KK}^2} (\Delta_{W^\pm}^L)_{bt} \\
&\quad + O\left(\frac{v^3}{\Lambda_{KK}^3}\right), \\
(\xi_{W^\pm}^{R(-1/3)})_{b,t} &= \frac{v^2}{2\Lambda_{KK}^2} (\Delta_{W^\pm}^R)_{bt} + O\left(\frac{v^3}{\Lambda_{KK}^3}\right), \\
(\xi_{W_{H(2n-1)}^\pm}^{L(-1/3)})_{b,t} &= \frac{4\sqrt{2}\pi}{kr\epsilon} \sum_{i=1}^3 (D_L^{(0)})_{bi}^\dagger \int_\epsilon^1 dt \chi_{(++)}^{W_L}(y_{(++)}^{W_L(n)}, t) [f_{(++)}^{L,c_B^i}(0, t)]^2 (U_L^{(0)})_{it} \\
&\quad + O\left(\frac{v^2}{\Lambda_{KK}^2}\right), \quad (\alpha \geq 4), \\
(\xi_{W_{H(2n-1)}^\pm}^{L(-1/3)})_{\alpha,t} &= \frac{4\sqrt{2}\pi}{kr\epsilon} \sum_{i=1}^3 \sum_{n'=1}^\infty \delta_{\alpha(9n'-6+i)} \int_\epsilon^1 dt \chi_{(++)}^{W_L}(y_{(++)}^{W_L(n)}, t) \\
&\quad \times [f_{(++)}^{L,c_B^i}(y_{(\pm\pm)}^{c_B^i(n')}, t)] [f_{(++)}^{L,c_B^i}(0, t)] (U_L^{(0)})_{it} + O\left(\frac{v^2}{\Lambda_{KK}^2}\right), \quad (\alpha \geq 4) \quad (90)
\end{aligned}$$

with

$$\begin{aligned}
(\Delta_{W^\pm}^L)_{bt} &= - \sum_{i,j,k=1}^3 (D_L^{(0)})_{bi}^\dagger f_{(++)}^{L,c_B^i}(0, 1) \{ 2Y_{ik}^{d\dagger} [\Sigma_{(\pm\mp)}^{R,c_T^k}(1, 1)] Y_{kj}^d \\
&\quad + Y_{ik}^{d\dagger} [\Sigma_{(\mp\mp)}^{R,c_T^k}(1, 1)] Y_{kj}^d + Y_{ik}^{u\dagger} [\Sigma_{(\mp\mp)}^{R,c_S^k}(1, 1)] Y_{kj}^u \} f_{(++)}^{L,c_B^j}(0, 1) (U_L^{(0)})_{jt} \\
&\quad - \frac{8\pi e^2}{s_W^2} \sum_{i=1}^3 (D_L^{(0)})_{ti}^\dagger \left\{ \frac{1}{kr\epsilon} \int_\epsilon^1 dt [f_{(++)}^{L,c_B^i}(0, t)]^2 [\Sigma_{(++)}^G(t, 1)] (U_L^{(0)})_{ib} \right\}, \\
(\Delta_{W^\pm}^R)_{bt} &= \sum_{i,j,k=1}^3 (D_R^{(0)})_{bi}^\dagger f_{(++)}^{R,c_S^i}(0, 1) Y_{ik}^d [\Sigma_{(\pm\pm)}^{L,c_B^k}(1, 1)] Y_{kj}^{d\dagger} f_{(++)}^{R,c_T^j}(0, 1) (U_R^{(0)})_{jt}. \quad (91)
\end{aligned}$$

Similarly, the nontrivial couplings involving in Fig.(a1),(a2) with charged  $5/3$  quarks are approximated by



$$\begin{aligned}
(\xi_{W_{H(2n)}^\pm}^{L(5/3)})_{\alpha,t} &= \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 \sum_{n'=1}^{\infty} \delta_{\alpha(9(n'-1)-3+i)} \int_{\epsilon}^1 dt \chi_{(-+)}^{W_R}(y_{(-+)}^{W_R(n)}, t) \\
&\quad \times [f_{(-+)}^{L,c_B^i}(y_{(\mp\pm)}^{c_B^i(n')}, t)] [f_{(++)}^{L,c_B^i}(0, t)] (U_L^{(0)})_{it} + O(\frac{v^2}{\Lambda_{KK}^2}), \quad (\alpha \geq 4), \quad (92)
\end{aligned}$$

The nontrivial couplings involving in Fig.(b1),(b2) with charged  $-1/3$  quarks are approximated by

$$\begin{aligned}
(\eta_{G^\pm}^{L(-1/3)})_{b,t} &= \frac{e}{\sqrt{2}s_w} \left\{ \frac{m_t}{m_W} (V_{CKM}^{(0)})_{bt}^\dagger - \frac{(\delta M^u)_{33}^*}{m_W} (V_{CKM}^{(0)})_{bt}^\dagger - \frac{\pi m_t m_W}{\Lambda_{KK}^2} [\{\Sigma_{(++)}^G(1, 1)\} \right. \\
&\quad + \{\Sigma_{(-+)}^G(1, 1)\}] (V_{CKM}^{(0)})_{bt}^\dagger + \sum_{i=1}^3 \left[ \frac{m_{u_i}}{m_W} (\delta Z_R^u V_{CKM}^{(0)})_{bt}^\dagger \right. \\
&\quad + \left. \frac{m_t}{m_W} (V_{CKM}^{(0)} \delta Z_L^d)_{bt}^\dagger \right] + \frac{v^2}{4\Lambda_{KK}^2} (\Delta_{G^\pm}^L)_{bt} \left. \right\} + O(\frac{v^3}{\Lambda_{KK}^3}) \\
(\eta_{G^\pm}^{R(-1/3)})_{b,t} &= \frac{e}{\sqrt{2}s_w} \left\{ (V_{CKM}^{(0)})_{bt}^\dagger \frac{m_b}{m_W} - \frac{(\delta M^d)_{33}^*}{m_W} (V_{CKM}^{(0)})_{bt}^\dagger - \frac{\pi m_t m_W}{\Lambda_{KK}^2} [\{\Sigma_{(++)}^G(1, 1)\} \right. \\
&\quad + \{\Sigma_{(-+)}^G(1, 1)\}] (V_{CKM}^{(0)})_{bt}^\dagger + \sum_{i=1}^3 \left[ \frac{m_b}{m_W} (\delta Z_R^d V_{CKM}^{(0)})_{bt}^\dagger \right. \\
&\quad + \left. \frac{m_{d_i}}{m_W} (V_{CKM}^{(0)} \delta Z_L^d)_{bt}^\dagger \right] + \frac{v^2}{4\Lambda_{KK}^2} (\Delta_{G^\pm}^R)_{bt} \left. \right\} + O(\frac{v^3}{\Lambda_{KK}^3}) \\
(\eta_{G^\pm}^{L(-1/3)})_{\alpha,t} &= \sum_{i,j=1}^3 \sum_{n=1}^{\infty} \delta_{\alpha,(9n+i)} f_{(++)}^{R,c_T^i}(y_{(\mp\mp)}^{c_T^i(n)}, 1) Y_{ij}^{d\dagger} f_{(++)}^{L,c_B^j}(0, 1) (U_L^{(0)})_{j,t} + O(\frac{v^2}{\Lambda_{KK}^2}) \\
(\eta_{G^\pm}^{R(-1/3)})_{\alpha,t} &= \sum_{i,j=1}^3 \sum_{n=1}^{\infty} \delta_{\alpha,(9n-6+i)} f_{(++)}^{L,c_B^i}(y_{(\pm\pm)}^{c_B^i(n)}, 1) Y_{ij}^u f_{(++)}^{R,c_S^j}(0, 1) (U_R^{(0)})_{j,t} + O(\frac{v^2}{\Lambda_{KK}^2}) \quad (93)
\end{aligned}$$

with

$$\begin{aligned}
(\Delta_{G^\pm}^L)_{bt} &= - \sum_{i,j,k,l=1}^3 \frac{m_{ul}}{m_W} (V_{CKM}^{(0)})_{bl}^\dagger (U_R^{(0)})_{li}^\dagger f_{(++)}^{R,c_S^i}(0,1) Y_{ik}^u [\{\Sigma_{(\pm\pm)}^{L,c_B^k}(1,1)\} + \{\Sigma_{(\mp\pm)}^{L,c_B^k}(1,1)\}] \\
&\quad \times Y_{kj}^{u\dagger} f_{(++)}^{R,c_S^j}(0,1) (U_R^{(0)})_{jt} \\
&\quad - \sum_{i,j,k,l=1}^3 \frac{m_t}{m_W} (V_{CKM}^{(0)})_{bl}^\dagger (U_L^{(0)})_{li}^\dagger f_{(++)}^{L,c_B^i}(0,1) Y_{ik}^{d\dagger} [\{\Sigma_{(\pm\mp)}^{R,c_T^k}(1,1)\} + \{\Sigma_{(\mp\mp)}^{R,c_T^k}(1,1)\}] \\
&\quad \times Y_{kj}^d f_{(++)}^{L,c_B^j}(0,1) (U_L^{(0)})_{jt}, \\
(\Delta_{G^\pm}^R)_{bt} &= - \sum_{i,j,k,l=1}^3 \frac{m_{di}}{m_W} (D_R^{(0)})_{bk}^\dagger f_{(++)}^{R,c_T^k}(0,1) Y_{kl}^d [\Sigma_{(\pm\pm)}^{L,c_B^l}(1,1)] \\
&\quad \times Y_{lj}^{d\dagger} f_{(++)}^{R,c_S^j}(0,1) (D_R^{(0)})_{ji} (V_{CKM}^{(0)})_{it}^\dagger \\
&\quad - \sum_{i,j,k,l=1}^3 \frac{m_b}{m_W} (D_L^{(0)})_{bk}^\dagger f_{(++)}^{L,c_B^k}(0,1) [Y_{kl}^{d\dagger} \{\Sigma_{(\pm\mp)}^{R,c_T^l}(1,1)\} Y_{lj}^d \\
&\quad + Y_{kl}^{u\dagger} \{\Sigma_{(\mp\mp)}^{R,c_T^l}(1,1)\} Y_{lj}^u] f_{(++)}^{L,c_B^j}(0,1) (D_L^{(0)})_{ji} (V_{CKM}^{(0)})_{it}^\dagger.
\end{aligned} \tag{94}$$

The nontrivial couplings involving in Fig.(b1),(b2) with charged 5/3 quarks are approximated by

$$\begin{aligned}
(\eta_{G^\pm}^{L(5/3)})_{\alpha,t} &= \sum_{i,j=1}^3 \sum_{n=1}^\infty \{ \delta_{\alpha,(9n-3+i)} f_{(-+)}^{R,c_T^i}(y_{(\pm\mp)}^{c_T^{(n)}}, 1) Y_{ij}^{d\dagger} + \delta_{\alpha,(9n-6+i)} f_{(-+)}^{R,c_T^i}(y_{(\pm\mp)}^{c_T^{(n)}}, 1) Y_{ij}^{d\dagger} \} \\
&\quad \times f_{(++)}^{L,c_B^j}(0,1) (U_L^{(0)})_{j,t} + O(\frac{v^2}{\Lambda_{KK}^2}) \\
(\eta_{G^\pm}^{R(5/3)})_{\alpha,t} &= - \sum_{i,j=1}^3 \sum_{n=1}^\infty \delta_{\alpha,(9(n-1)+i)} f_{(-+)}^{L,c_B^i}(y_{(\mp\pm)}^{c_B^{(n)}}, 1) Y_{ij}^u f_{(++)}^{R,c_S^j}(0,1) (U_R^{(0)})_{j,t} + O(\frac{v^2}{\Lambda_{KK}^2})
\end{aligned} \tag{95}$$

As the FCNC transitions are mediated by the massive neutral gauge bosons  $Z$ ,  $Z_{H_\alpha}$ ,  $\gamma_{(n)}$  in Fig.(c), relevant couplings are expanded according  $v^2/\Lambda_{KK}^2$  as

$$\begin{aligned}
(\xi_Z^{L(2/3)})_{c,t} &= \left(\frac{3-4s_w^2}{6s_w c_w}\right) \left[ \delta_{ct} + (\delta Z_L^u)_{ct}^\dagger + (\delta Z_L^u)_{ct} + \frac{v^2}{2\Lambda_{KK}^2} (\Delta_Z^{L2/3})_{ct} \right] + O\left(\frac{v^3}{\Lambda_{KK}^3}\right), \\
(\xi_Z^{R(2/3)})_{c,t} &= -\frac{2}{3} \frac{s_w}{c_w} \left[ \delta_{ct} + (\delta Z_R^u)_{ct}^\dagger + (\delta Z_R^u)_{ct} + \frac{v^2}{2\Lambda_{KK}^2} (\Delta_Z^{R2/3})_{ct} \right] + O\left(\frac{v^3}{\Lambda_{KK}^3}\right), \\
(\xi_{Z_{H(2n-1)}}^{L(2/3)})_{c,t} &= \left(\frac{3-4s_w^2}{6s_w c_w}\right) \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 (U_L^{(0)})_{ci}^\dagger \int_\epsilon^1 dt \chi_{(++)}^Z(y_{(++)}^{Z(n)}, t) \\
&\quad \times [f_{(++)}^{L,c_B^i}(0, t)]^2 (U_L^{(0)})_{it} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{Z_{H(2n)}}^{L(2/3)})_{c,t} &= -\frac{3-2s_w^2}{6s_w c_w \sqrt{1-2s_w^2}} \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 (U_L^{(0)})_{ci}^\dagger \int_\epsilon^1 dt \chi_{(-+)}^{Z_X}(y_{(-+)}^{Z_X(n)}, t) \\
&\quad \times [f_{(++)}^{L,c_B^i}(0, t)]^2 (U_L^{(0)})_{it} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{Z_{H(2n-1)}}^{R(2/3)})_{c,t} &= -\frac{2}{3} \frac{s_w}{c_w} \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 (U_R^{(0)})_{ci}^\dagger \int_\epsilon^1 dt \chi_{(++)}^Z(y_{(++)}^{Z(n)}, t) \\
&\quad \times [f_{(++)}^{R,c_S^i}(0, t)]^2 (U_R^{(0)})_{it} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{Z_{H(2n)}}^{R(2/3)})_{c,t} &= -\frac{2s_w}{3c_w \sqrt{1-2s_w^2}} \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 (U_R^{(0)})_{ci}^\dagger \int_\epsilon^1 dt \chi_{(-+)}^{Z_X}(y_{(-+)}^{Z_X(n)}, t) \\
&\quad \times [f_{(++)}^{R,c_S^i}(0, t)]^2 (U_R^{(0)})_{it} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{Z_{H(2n-1)}}^{L(2/3)})_{\alpha,t} &= \left(\frac{3-4s_w^2}{6s_w c_w}\right) \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 \sum_{n'=1}^\infty \delta_{\alpha(15n'-12+i)} \int_\epsilon^1 dt \chi_{(++)}^Z(y_{(++)}^{Z(n)}, t) \\
&\quad \times [f_{(++)}^{L,c_B^i}(0, t)] [f_{(++)}^{L,c_B^i}(y_{(\pm\pm)}^{c_B(n')}, t)] (U_L^{(0)})_{it} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{Z_{H(2n)}}^{L(2/3)})_{\alpha,t} &= -\frac{3-2s_w^2}{6s_w c_w \sqrt{1-2s_w^2}} \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 \sum_{n'=1}^\infty \delta_{\alpha(15n'-12+i)} \int_\epsilon^1 dt \chi_{(-+)}^{Z_X}(y_{(-+)}^{Z_X(n)}, t) \\
&\quad \times [f_{(++)}^{L,c_B^i}(0, t)] [f_{(++)}^{L,c_B^i}(y_{(\pm\pm)}^{c_B(n')}, t)] (U_L^{(0)})_{it} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{Z_{H(2n-1)}}^{R(2/3)})_{\alpha,t} &= -\frac{2}{3} \frac{s_w}{c_w} \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 \sum_{n'=1}^\infty \delta_{\alpha(15n'+i)} \int_\epsilon^1 dt \chi_{(++)}^Z(y_{(++)}^{Z(n)}, t) \\
&\quad \times [f_{(++)}^{R,c_S^i}(0, t)] [f_{(++)}^{R,c_S^i}(y_{(\pm\pm)}^{c_S(n')}, t)] (U_R^{(0)})_{it} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{Z_{H(2n)}}^{R(2/3)})_{\alpha,t} &= -\frac{2s_w}{3c_w \sqrt{1-2s_w^2}} \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 \sum_{n'=1}^\infty \delta_{\alpha(15n'+i)} \int_\epsilon^1 dt \chi_{(-+)}^{Z_X}(y_{(-+)}^{Z_X(n)}, t) \\
&\quad \times [f_{(++)}^{R,c_S^i}(0, t)] [f_{(++)}^{R,c_S^i}(y_{(\pm\pm)}^{c_S(n')}, t)] (U_R^{(0)})_{it} + O\left(\frac{v}{\Lambda_{KK}}\right),
\end{aligned}$$

$$\begin{aligned}
(\xi_{\gamma(n)}^{L(2/3)})_{c,t} &= \frac{2}{3} \frac{4\sqrt{2}\pi}{kr\epsilon} \sum_{i=1}^3 (U_L^{(0)})_{c,i}^\dagger \int_\epsilon^1 dt \chi_{(++)}^A(y_{(++)}^{A(n)}, t) [f_{(++)}^{L,c_B^i}(0, t)]^2 (U_L^{(0)})_{i,t} \\
&\quad + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{\gamma(n)}^{R(2/3)})_{c,t} &= \frac{2}{3} \frac{4\sqrt{2}\pi}{kr\epsilon} \sum_{i=1}^3 (U_R^{(0)})_{c,i}^\dagger \int_\epsilon^1 dt \chi_{(++)}^A(y_{(++)}^{A(n)}, t) [f_{(++)}^{R,c_S^i}(0, t)]^2 (U_R^{(0)})_{i,t} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{\gamma(n)}^{L(2/3)})_{\alpha,t} &= \frac{2}{3} \frac{4\sqrt{2}\pi}{kr\epsilon} \sum_{i=1}^3 \sum_{n'=1}^\infty \delta_{\alpha,(15n'-12+i)} \int_\epsilon^1 dt \chi_{(++)}^A(y_{(++)}^{A(n)}, t) \{f_{(++)}^{L,c_B^i}(0, t) \\
&\quad \times f_{(++)}^{L,c_B^i}(y_{(\pm\pm)}^{c_B^{i(n')}}, t)\} (U_L^{(0)})_{i,t} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{\gamma(n)}^{R(2/3)})_{\alpha,t} &= \frac{2}{3} \frac{4\sqrt{2}\pi}{kr\epsilon} \sum_{i=1}^3 \sum_{n'=1}^\infty \delta_{\alpha,(15n'+i)} \int_\epsilon^1 dt \chi_{(++)}^A(y_{(++)}^{A(n)}, t) \{f_{(++)}^{R,c_S^i}(0, t) \\
&\quad \times f_{(++)}^{R,c_S^i}(y_{(\mp\mp)}^{c_S^{i(n')}}, t)\} (U_R^{(0)})_{i,b} + O\left(\frac{v}{\Lambda_{KK}}\right). \tag{96}
\end{aligned}$$

Here the short-cut notations  $\Delta_Z^L$ ,  $\Delta_Z^R$  are defined by

$$\begin{aligned}
(\Delta_Z^{L2/3})_{ct} &= \sum_{i,j,k=1}^3 (U_L^{(0)})_{ci}^\dagger f_{(++)}^{L,c_B^i}(0, 1) \{Y_{ik}^d(\frac{6}{4s_w^2-3}[\Sigma_{(\pm\mp)}^{R,c_T^k}(1, 1)]) Y_{kj}^{d\dagger} \\
&\quad + Y_{ik}^u(\frac{3}{4s_w^2-3}[\Sigma_{(\mp\mp)}^{R,c_S^k}(1, 1)]) Y_{kj}^{u\dagger}\} \times f_{(++)}^{L,c_B^j}(0, 1) (U_L^{(0)})_{jt} \\
&\quad - \frac{4\pi e^2}{s_w^2 c_w^2 kr\epsilon} \sum_{i=1}^3 (U_L^{(0)})_{ci}^\dagger \left( \int_\epsilon^1 dt \{[\Sigma_{(++)}^G(t, 1)] \right. \\
&\quad \left. + \frac{2s_w^2-3}{3-4s_w^2}[\Sigma_{(-+)}^G(t, 1)]\} [f_{(++)}^{L,c_B^i}(0, t)]^2 \right) (U_L^{(0)})_{it}, \\
(\Delta_Z^{R2/3})_{ct} &= \frac{3}{4s_w^2} \sum_{i,j,k=1}^3 (U_R^{(0)})_{ci}^\dagger f_{(++)}^{R,c_S^i}(0, 1) Y_{ik}^{u\dagger} \{[\Sigma_{(\mp\pm)}^{L,c_B^k}(1, 1)] \\
&\quad - [\Sigma_{(\pm\pm)}^{L,c_B^k}(1, 1)]\} Y_{kj}^u f_{(++)}^{R,c_S^j}(0, 1) (U_R^{(0)})_{jt} \\
&\quad - \frac{4\pi e^2}{s_w^2 c_w^2 kr\epsilon} \sum_{i,j=1}^3 (U_R^{(0)})_{ci}^\dagger \left( \int_\epsilon^1 dt \{[\Sigma_{(++)}^G(t, 1)] + s_w[\Sigma_{(-+)}^G(t, 1)]\} \right. \\
&\quad \left. \times [f_{(++)}^{L,c_S^i}(0, t)]^2 \right) (U_R^{(0)})_{it} \tag{97}
\end{aligned}$$

In Fig.(d1) and (d2), the KK exciting modes of gluon also mediate the FCNC transitions and the relevant couplings are approximated by

$$\begin{aligned}
(\xi_{g(n)}^{L(2/3)})_{c,t} &= \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 (U_L^{(0)})_{c,i}^\dagger \int_\epsilon^1 dt \chi_{(++)}^g(y_{(++)}^{g(n)}, t) [f_{(++)}^{L,c_B^i}(0, t)]^2 (U_L^{(0)})_{i,t} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{g(n)}^{R(2/3)})_{c,t} &= \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 (U_R^{(0)})_{c,i}^\dagger \int_\epsilon^1 dt \chi_{(++)}^g(y_{(++)}^{g(n)}, t) [f_{(++)}^{R,c_S^i}(0, t)]^2 (U_R^{(0)})_{i,t} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{g(n)}^{L(2/3)})_{\alpha,t} &= \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 \sum_{n'=1}^\infty \delta_{\alpha,(15n'-12+i)} \int_\epsilon^1 dt \chi_{(++)}^g(y_{(++)}^{g(n)}, t) \{f_{(++)}^{L,c_B^i}(0, t) \\
&\quad \times f_{(++)}^{L,c_B^i}(y_{(\pm\pm)}^{c_B^i(n')}, t)\} (U_L^{(0)})_{i,t} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\xi_{g(n)}^{R(2/3)})_{\alpha,t} &= \frac{4\sqrt{2\pi}}{kr\epsilon} \sum_{i=1}^3 \sum_{n'=1}^\infty \delta_{\alpha,(15n'+i)} \int_\epsilon^1 dt \chi_{(++)}^g(y_{(++)}^{g(n)}, t) \{f_{(++)}^{R,c_S^i}(0, t) \\
&\quad \times f_{(++)}^{R,c_S^i}(y_{(\mp\mp)}^{c_S^i(n')}, t)\} (U_R^{(0)})_{i,b} + O\left(\frac{v}{\Lambda_{KK}}\right). \tag{98}
\end{aligned}$$

Finally, In Fig.(e), the relevant FCNC couplings mediated by neutral Higgs and Goldstone are

$$\begin{aligned}
(\eta_{H_0}^{L(2/3)})_{c,t} &= -\frac{e}{\sqrt{2}s_w} \left\{ \frac{m_t}{m_W} \delta_{ct} - \frac{(\delta M^u)_{33}}{m_W} \delta_{ct} + \frac{m_t}{m_W} (\delta Z_R^u)_{ct}^\dagger + \frac{m_c}{m_W} (\delta Z_L^u)_{ct} \right. \\
&\quad \left. - \delta_{ct} \frac{\pi m_t m_W}{\Lambda_{KK}^2} [\{\Sigma_{(++)}^G(1,1)\} + \{\Sigma_{(-+)}^G(1,1)\}] \right. \\
&\quad \left. + \frac{v^2}{4\Lambda_{KK}^2} \left[ \frac{m_t}{m_W} (\Delta_{H_0}^{(1)2/3})_{ct} + \frac{m_c}{m_W} (\Delta_{H_0}^{(2)2/3})_{ct} \right] \right\} + O\left(\frac{v^3}{\Lambda_{KK}^3}\right), \\
(\eta_{H_0}^{R(2/3)})_{c,t} &= (\eta_{H_0}^{L(2/3)})_{c,t}^\dagger, \\
(\eta_{H_0}^{L(2/3)})_{\alpha,t} &= \sum_{i,j=1}^3 \sum_{n=1}^\infty \left\{ -\delta_{\alpha,(15n+i)} f_{(++)}^{R,c_S^i} (y_{(\mp\mp)}^{c_S^i(n)}, 1) Y_{ij}^{u\dagger} f_{(++)}^{L,c_B^j} (0,1) (U_L^{(0)})_{j,t} \right. \\
&\quad + \frac{1}{\sqrt{2}} \delta_{\alpha,(15n-9+i)} f_{(-+)}^{R,c_T^i} (y_{(\pm\mp)}^{c_T^i(n)}, 1) Y_{ij}^{d\dagger} f_{(++)}^{L,c_B^j} (0,1) (U_L^{(0)})_{j,t} \\
&\quad \left. - \frac{1}{\sqrt{2}} \delta_{\alpha,(15n-6+i)} f_{(-+)}^{R,c_T^i} (y_{(\pm\mp)}^{c_T^i(n)}, 1) Y_{ij}^{d\dagger} f_{(++)}^{L,c_B^j} (0,1) (U_L^{(0)})_{j,t} \right\} + O\left(\frac{v}{\Lambda_{KK}}\right) \\
(\eta_{H_0}^{R(2/3)})_{\alpha,t} &= \sum_{i,j=1}^3 \sum_{n=1}^\infty \left\{ -\delta_{\alpha,(15n-12+i)} f_{(++)}^{L,c_B^i} (y_{(\pm\pm)}^{c_B^i(n)}, 1) Y_{ij}^{u\dagger} f_{(++)}^{R,c_S^j} (0,1) (U_R^{(0)})_{j,t} \right. \\
&\quad \left. + \sum_{n=1}^\infty \delta_{\alpha,(15n-3+i)} f_{(-+)}^{L,c_B^i} (y_{(\mp\pm)}^{c_B^i(n)}, 1) Y_{ij}^{u\dagger} f_{(++)}^{R,c_S^j} (0,1) (U_R^{(0)})_{j,t} \right\} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\eta_{G_0}^{L(2/3)})_{c,t} &= (\eta_{H_0}^{L(2/3)})_{c,t}, \\
(\eta_{G_0}^{R(2/3)})_{c,t} &= (\eta_{G_0}^{L(2/3)})_{c,t}^\dagger, \\
(\eta_{G_0}^{L(2/3)})_{\alpha,t} &= \sum_{i,j=1}^3 \left\{ \sum_{n=1}^\infty (-\delta_{\alpha,(15n+i)} f_{(++)}^{R,c_S^i} (y_{(\mp\mp)}^{c_S^i(n)}, 1) Y_{ij}^{u\dagger} f_{(++)}^{L,c_B^j} (0,1) (U_L^{(0)})_{j,t} \right. \\
&\quad + \sum_{n=1}^\infty \left( \frac{1}{\sqrt{2}} \delta_{\alpha,(15n-9+i)} f_{(-+)}^{R,c_T^i} (y_{(\pm\mp)}^{c_T^i(n)}, 1) Y_{ij}^{d\dagger} f_{(++)}^{L,c_B^j} (0,1) (U_L^{(0)})_{j,t} \right. \\
&\quad \left. - \sum_{n=1}^\infty \left( \frac{1}{\sqrt{2}} \delta_{\alpha,(15n-6+i)} f_{(-+)}^{R,c_T^i} (y_{(\pm\mp)}^{c_T^i(n)}, 1) Y_{ij}^{d\dagger} f_{(++)}^{L,c_B^j} (0,1) (U_L^{(0)})_{j,t} \right) \right\} + O\left(\frac{v}{\Lambda_{KK}}\right), \\
(\eta_{G_0}^{R(2/3)})_{\alpha,t} &= \sum_{i,j=1}^3 \sum_{n=1}^\infty \left\{ -\delta_{\alpha,(15n-3+i)} f_{(-+)}^{L,c_B^i} (y_{(\mp\pm)}^{c_B^i(n)}, 1) Y_{ij}^{u\dagger} f_{(++)}^{R,c_S^j} (0,1) (U_R^{(0)})_{j,t} \right. \\
&\quad \left. - \delta_{\alpha,(15n-12+i)} f_{(++)}^{L,c_B^i} (y_{(\pm\pm)}^{c_B^i(n)}, 1) Y_{ij}^{u\dagger} f_{(++)}^{R,c_S^j} (0,1) (U_R^{(0)})_{j,t} \right\} + O\left(\frac{v}{\Lambda_{KK}}\right). \tag{99}
\end{aligned}$$

where the abbreviations are given by

$$\begin{aligned}
(\Delta_{H_0}^{(1)2/3})_{ct} &= - \sum_{i,j,k,l=1}^3 (U_R^{(0)})_{ci}^\dagger f_{(++)}^{R,c_S^i}(0,1) Y_{ik}^{u\dagger} [\Sigma_{(\pm\pm)}^{L,c_B^k}(1,1)] Y_{kj}^u f_{(++)}^{R,c_S^j}(0,1) (U_L^{(0)})_{jt} \\
&\quad - \sum_{i,j,k,l=1}^3 (U_R^{(0)})_{ci}^\dagger f_{(++)}^{R,c_S^i}(0,1) Y_{ik}^{u\dagger} [\Sigma_{(\mp\pm)}^{L,c_B^k}(1,1)] Y_{kj}^u f_{(++)}^{R,c_S^j}(0,1) (U_L^{(0)})_{jt} \\
(\Delta_{H_0}^{(2)2/3})_{ct} &= - \sum_{i,j,k,l=1}^3 (U_R^{(0)})_{ci}^\dagger f_{(++)}^{L,c_B^i}(0,1) \{ Y_{ik}^d [2 \{ \Sigma_{(\pm\mp)}^{R,c_T^k}(1,1) \}] Y_{kj}^{d\dagger} + Y_{ik}^u [\{ \Sigma_{(\mp\mp)}^{R,c_S^k}(1,1) \}] Y_{kj}^{u\dagger} \} \\
&\quad \times f_{(++)}^{L,c_B^j}(0,1) (U_L^{(0)})_{jt} .
\end{aligned} \tag{100}$$

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